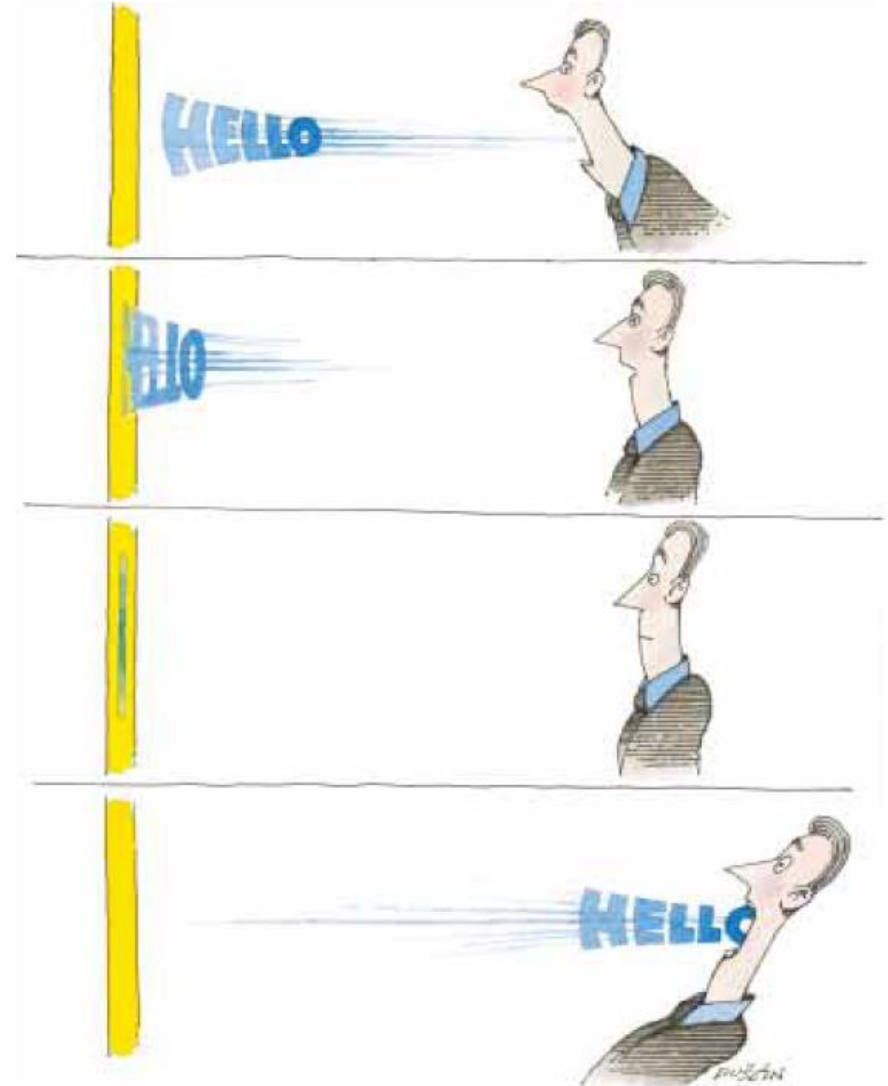


Full-Wave Seismic Holography Method RTH (Reverse Time Holography)

Prof. Gennady Erokhin



Seismic Challenge Problems

- ❖ Increasing the spatial resolution of seismic attributes, including for support high-precision technology of horizontal drilling

- ❖ Improving the quality of seismic data processing and of geologic results without increasing the cost of field work

- ❖ Building a ultra-high resolution velocity model using new mathematical approaches and adequate computing resources

- ❖ Possibility of direct exploration of hydrocarbon deposits based on joint analysis of seismic and GIS data using artificial intelligence methods

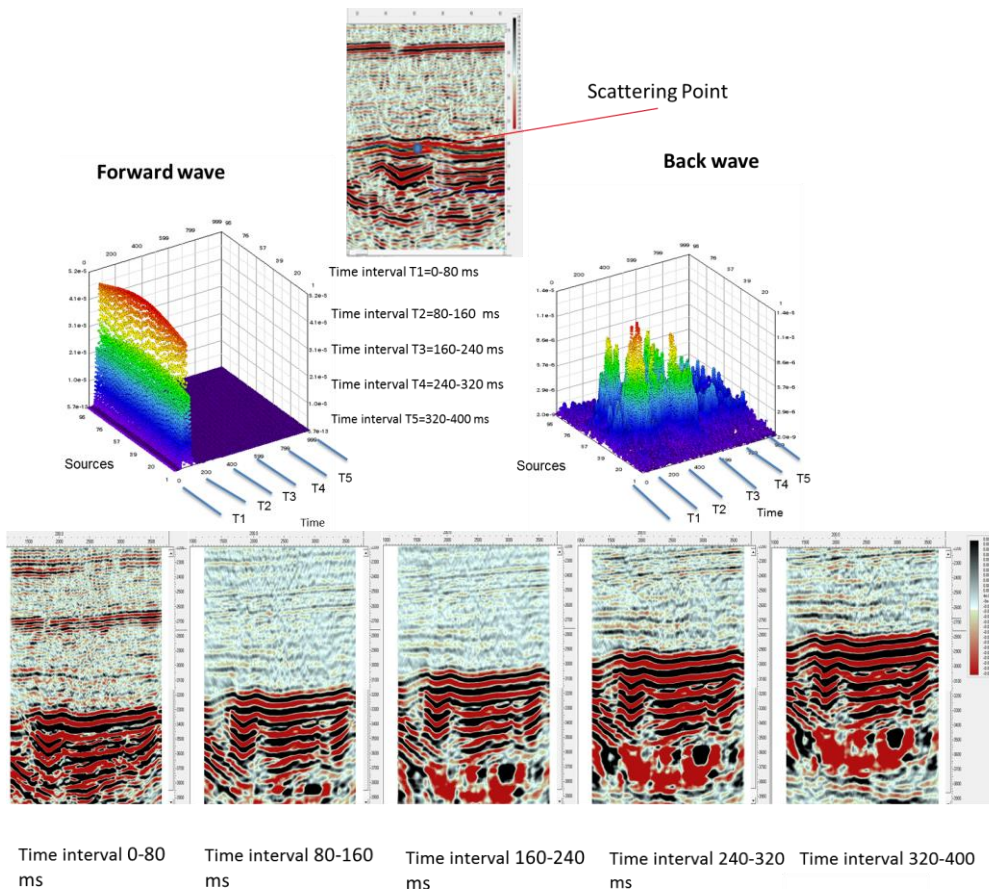
RTH Summary

Reverse Time Holography – RTH the new method of seismic data processing based on the reversal of the wave field in time and seismic holographic interferometry

Holography properties

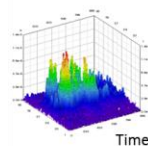
The RTH method is based on:

1. Theories of adjoint equations and reversing the wave field in time
2. Numerical spatial-temporal modeling of wave interference (holography)
3. Statistical accumulation of interference results (analogue - a set of photographic plates)
4. Filtering events of multidimensional statistical distribution
5. Statistical estimation of multidimensional distribution parameters
6. Multidimensional imaging technologies
7. Parallel Computing on Supercomputers
8. Extra large data processing



Layers shift 124 m, Shift Velocity 1550 m/s

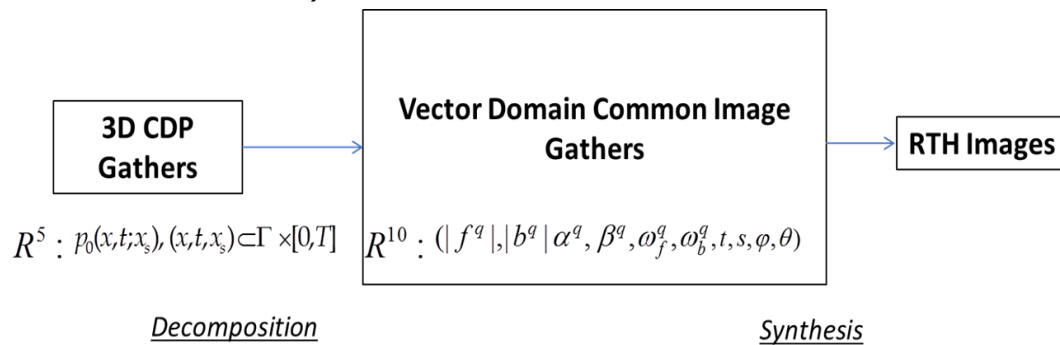
Migration velocity – 3100 m/s



Comparing RTH with Other Methods

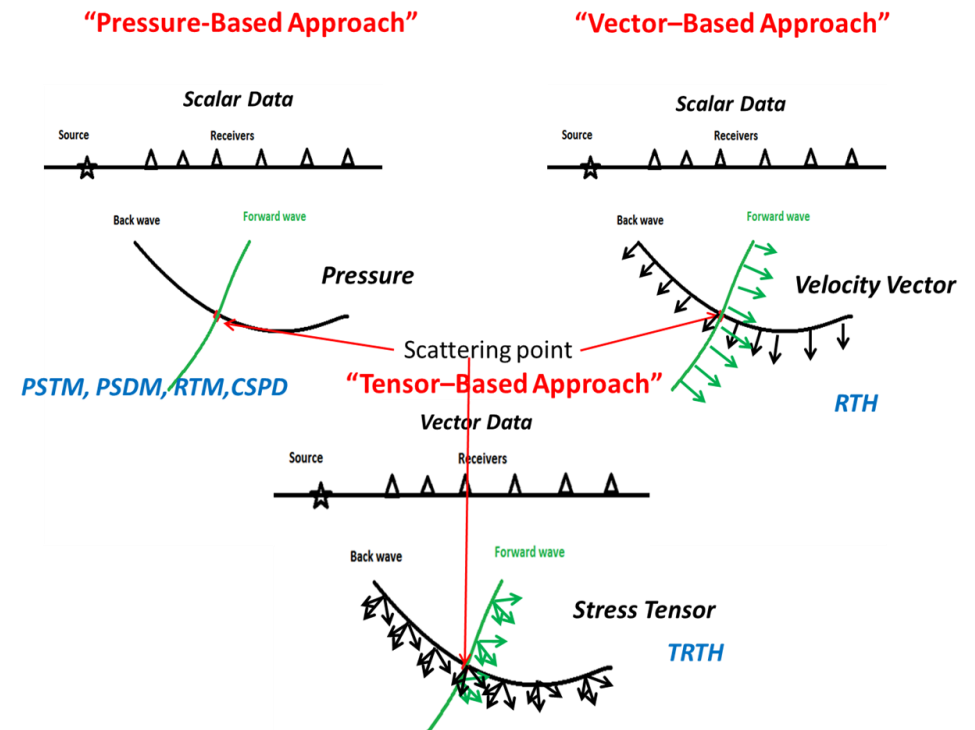
RTH processing workflow

Decomposition. Recalculation of the initial seismic data (for example, obtained using the Common Depth Point - CDP technology) into the data of a common image point of two vectors: the incident wave vector and the time-reversed "backward" wave (Vector Domain Common Image Gathers-VDCIG)



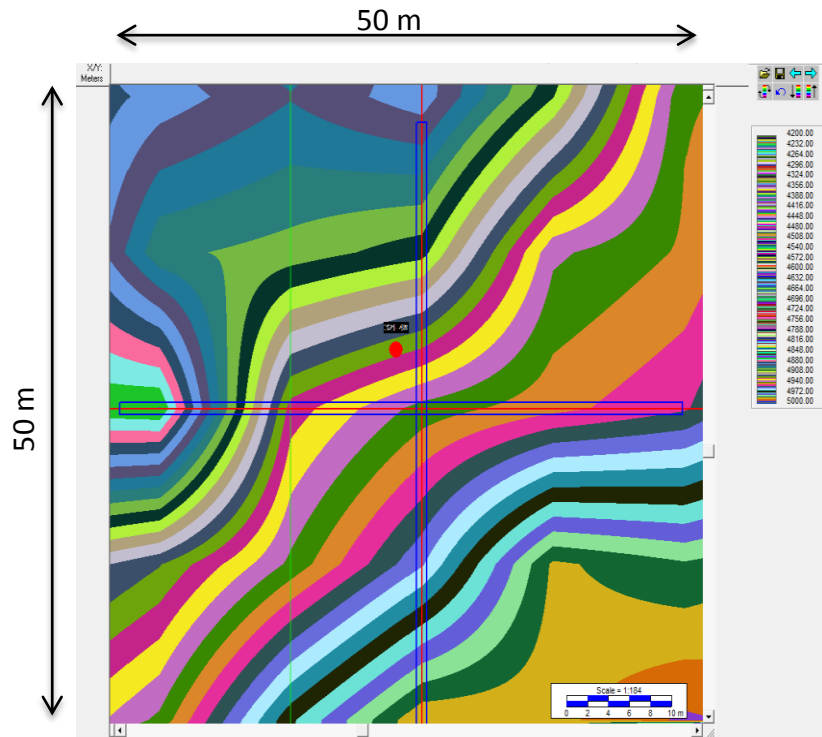
Synthesis. Statistical estimation of the VDCIG multidimensional distribution parameters in order to construct seismic attributes such as reflectors, diffractors, duplex waveforms, dip angles, scattering anisotropy, azimuthal anisotropy, AVO attributes, velocity, etc.

RTH and other seismic data processing methods

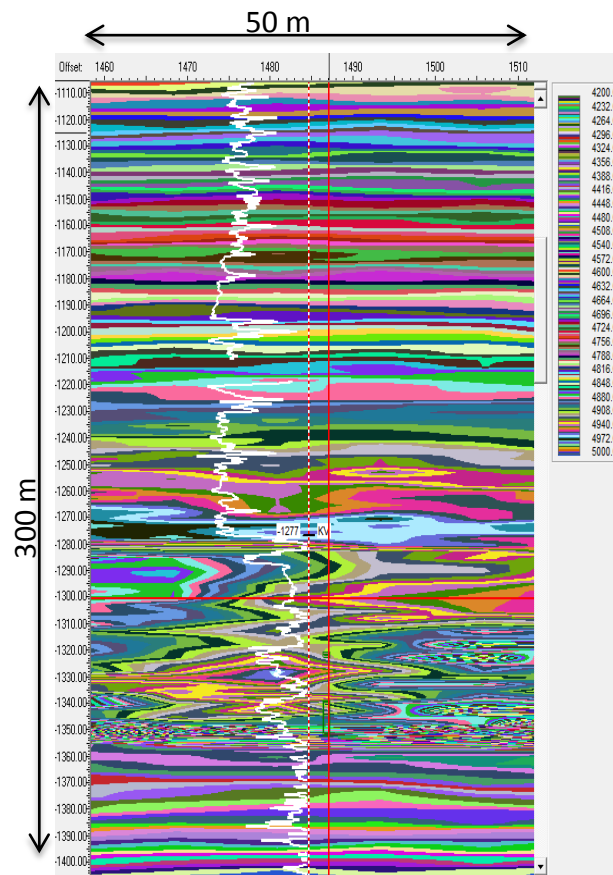




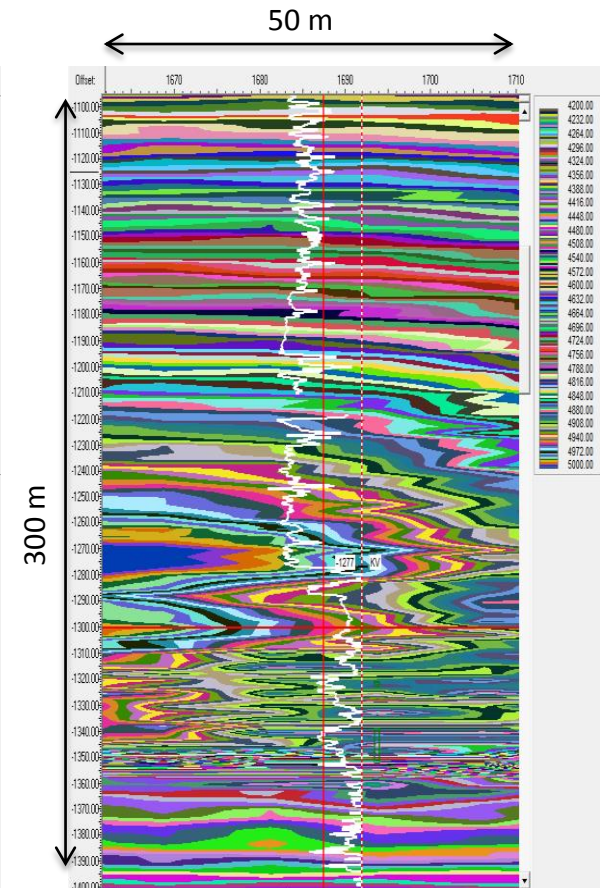
Velocity tomography with a spatial resolution of up to 1 meter



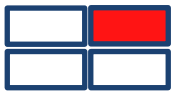
A horizontal slice of a 3D RTH velocity cube at a depth of 1300 m. The spatial size of the velocity voxel is 12.5x12.5x2.5 m. The dimension of the slice is 50x50 meters. Velocity scale - from 4200 to 5000 m / s. Eastern Siberia.



Vertical section of a RTH velocity cube. Inline, dimension is 50x300 m. Depths are from 1100 up to 1400 m. Velocity scale is from 4200 up to 5000 m/s. The white line is the Gamma Ray (GR) log.

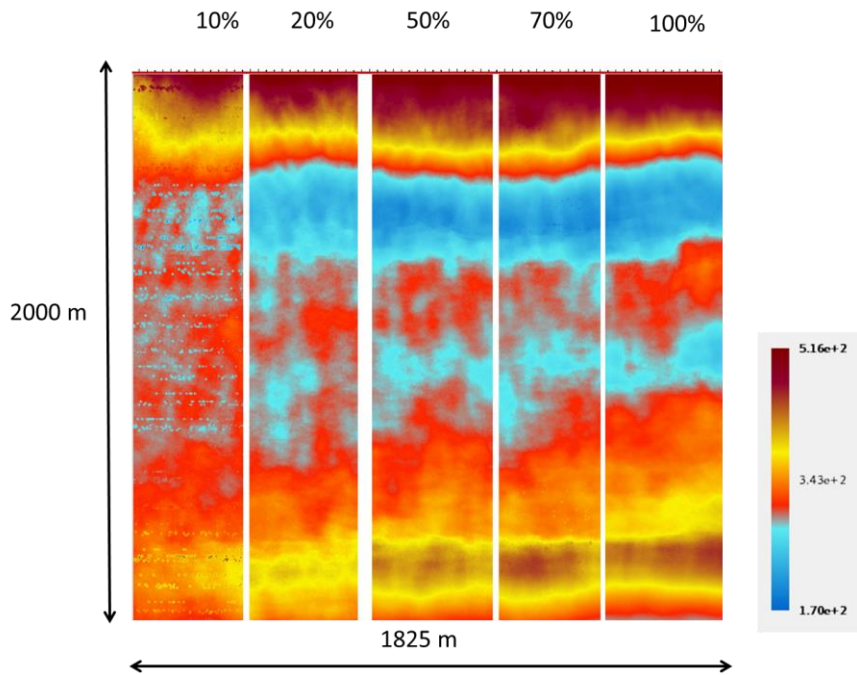


Vertical section of a RTH velocity cube. Crossline, dimension is 50x300 m. Depths are from 1100 up to 1400 m. Velocity scale is from 4200 up to 5000 m/s. The white line is the Gamma Ray (GR) log.

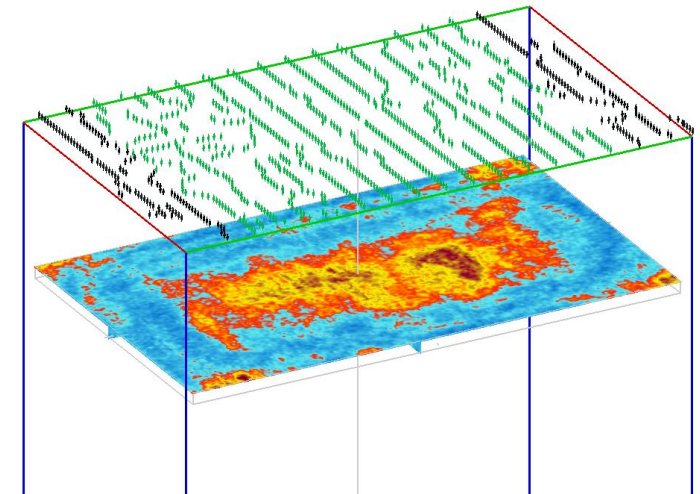
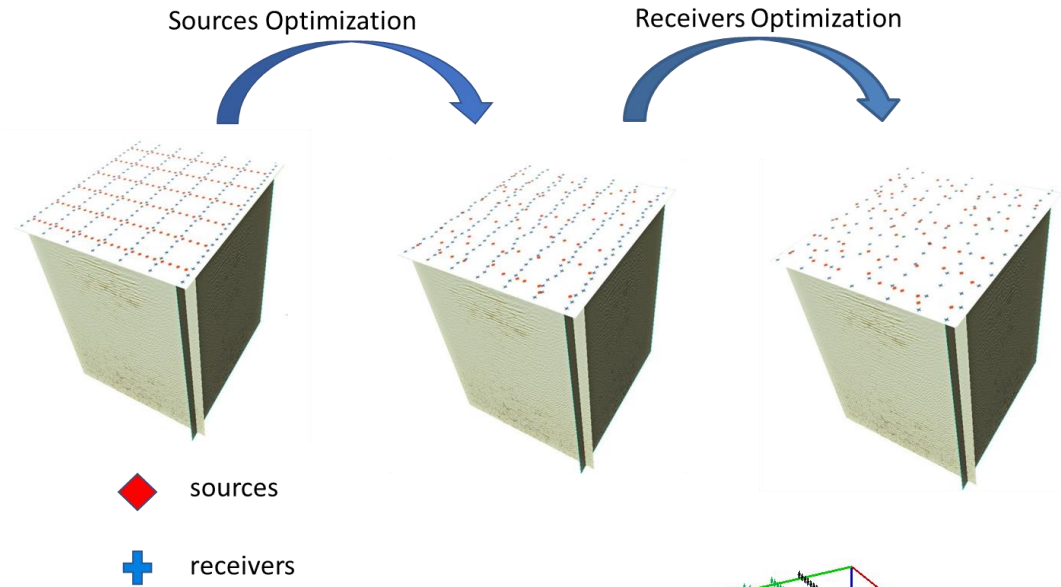


Stability RTH to sparse regular and sparse irregular seismic systems of receivers and sources

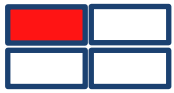
- Optimization of the cost of seismic exploration: the RTH method allows reducing the total number of sources and receivers during seismic exploration by at least two times.
- Reducing the number of sources does not reduce the quality of seismic processing
- The use of an irregular source system improves the quality of the attributes of the RTH method



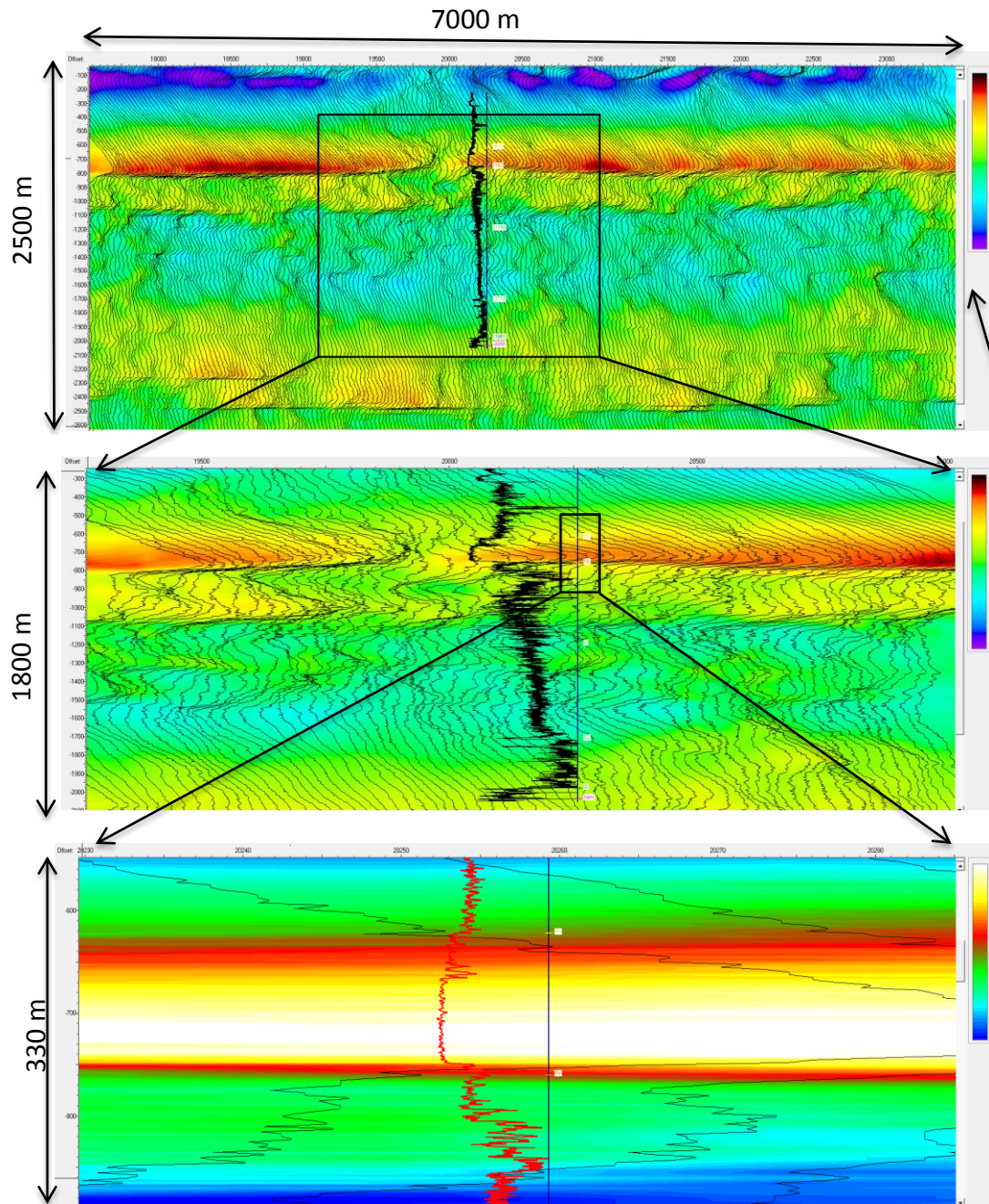
RTH diffraction for a different number of sources



The RTH method uses the true coordinates of the sources in space. Acoustic impedance slice. Baltic Syneclise, Curonian Depression.



Enhanced spatial resolution of seismic attributes

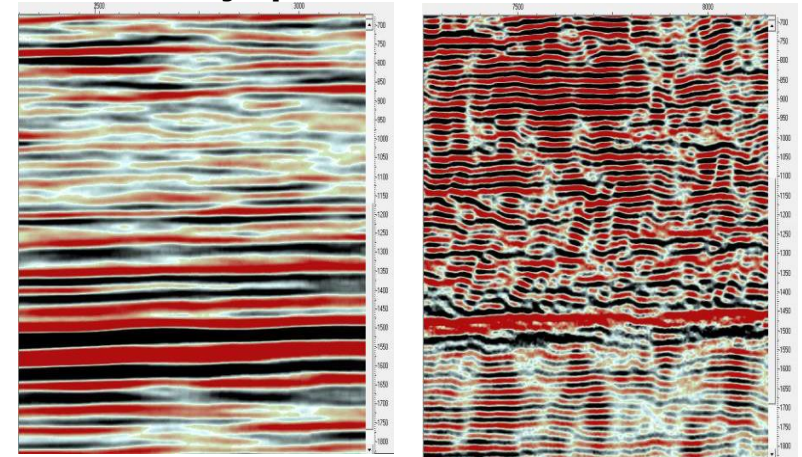


The spatial resolution of migration images is 3-6 times higher compared to conventional depth migration before summation

- Ultra high resolution (up to 1 meter)
- Ensuring efficient comparison of RTH attributes with GIS data due to the equivalent spatial resolution of RTH and GIS data

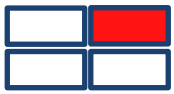
2D RTH velocity with overlay of the GR curve. The size of the velocity pixel is 12.5x2.5 m. The dimension of the region is 7.0x2.5 km. The red color of the palette is high-velocity deposits of Perm. Baltic Syncline, Curonian Depression.

Comparison of PSDM and RTH



**Kirchhoff
Depth Migration**

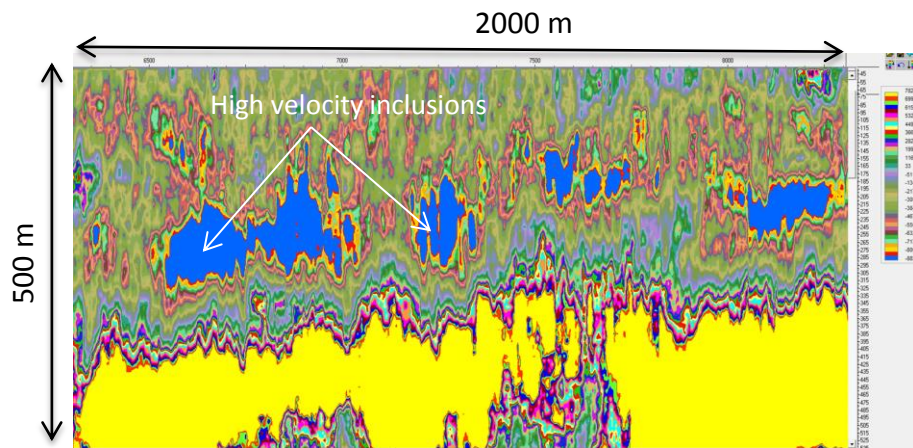
**RTH
Depth Migration**



Lack of near-surface layer problem

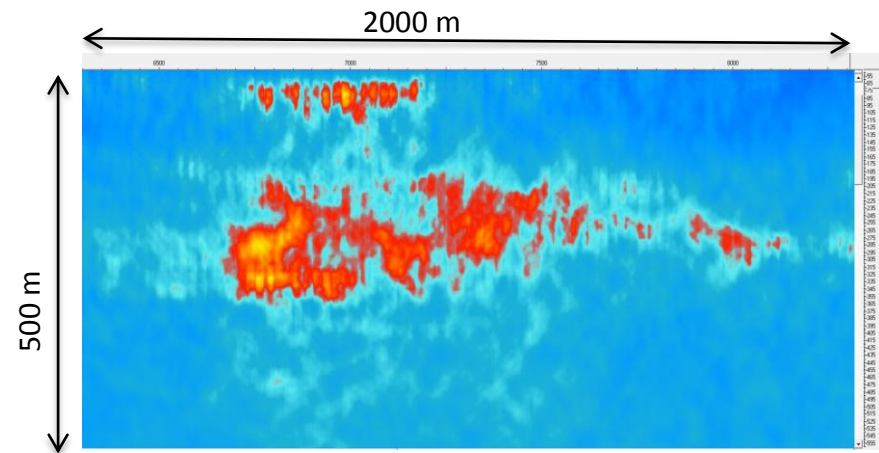
- The RTH method determines near-surface velocity and other attributes with high spatial resolution. Corrections for the near-surface layer for all attributes are not specifically taken into account. Western Siberia.

Near Surface Analysis

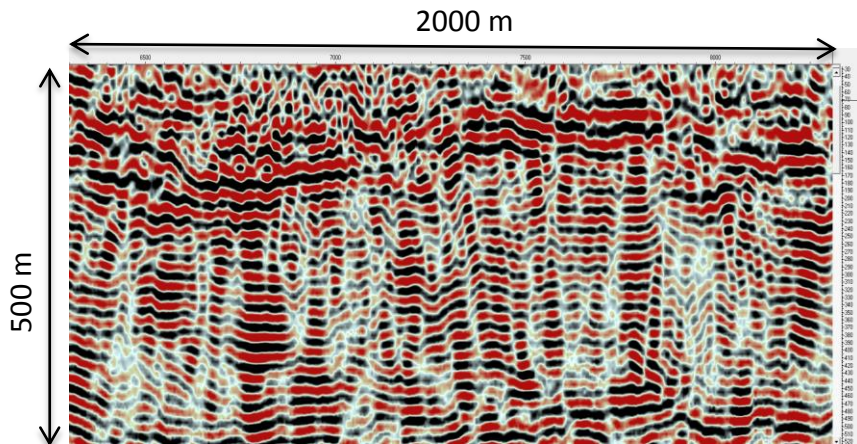


2D RTH Velocity, depth resolution 5m

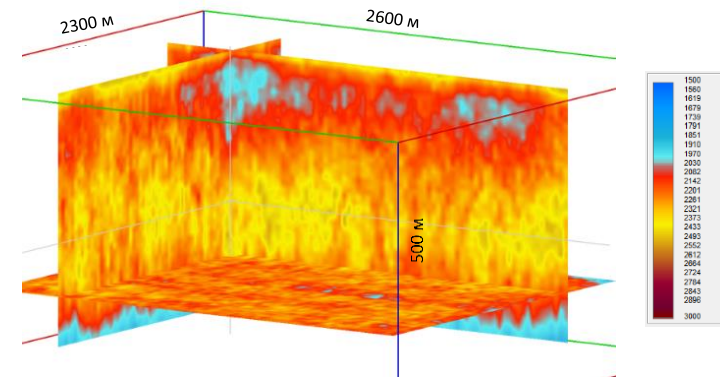
Near Surface Analysis



2D RTH Diffractivity, depth resolution 5m



2D RTH Reflectivity, depth resolution 5 m



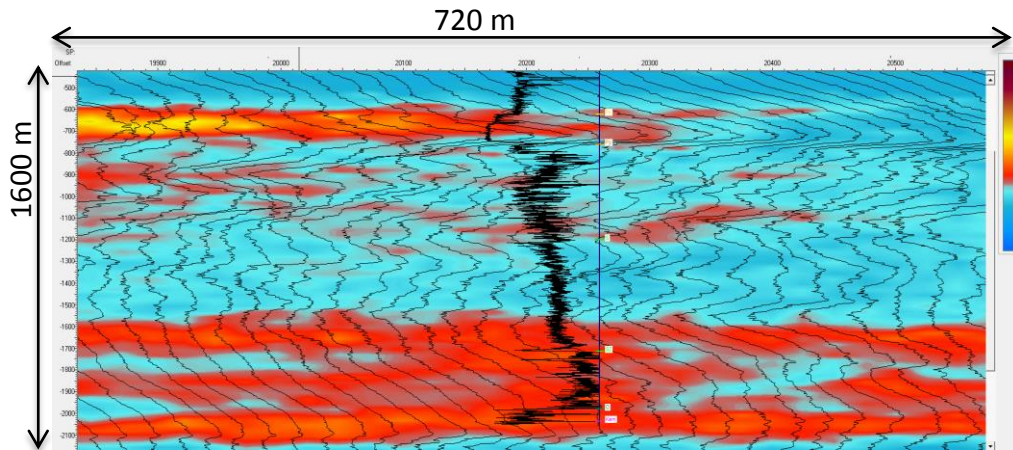
3D RTH Velocity, depth resolution 10 m



Building a high-resolution velocity model using new mathematical approaches and adequate computing resources

- RTH velocity calculation with significantly lower computational costs and significantly higher resolution than by high frequency Full Wave Inversion (FWI) method
- Calculation of over 20 new independent ultra-high spatial resolution seismic attributes

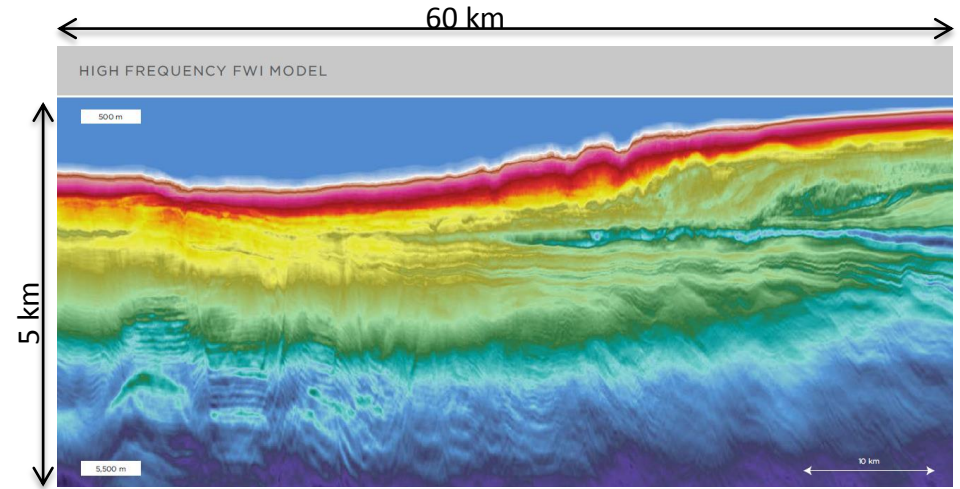
RTH Ultra High Resolution Diffractivity



HPC is 50 Teraflops. 2D RTH diffractors with GR curve and velocity (wiggle). The pixel size is 12.5x2.5 m. The dimension is 720x1600 m. Baltic Syncline, Curonian Depression.

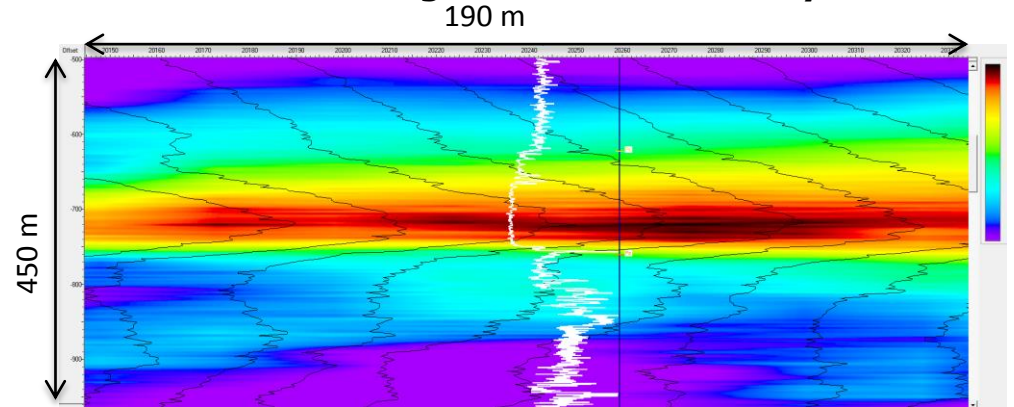
The cost of HPC 250 Petaflops is approximately 5000 times higher than the cost of HPC 50 Teraflops.

High frequency FWI velocity model by DUG Company

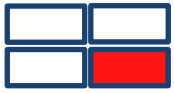


HPC is 250 Petaflops. Size is 60x5 km. Figure is from website of DUG Company : <https://dug.com/dug-geo/full-waveform-inversion-fwi/>.

RTH Ultra High Resolution Velocity

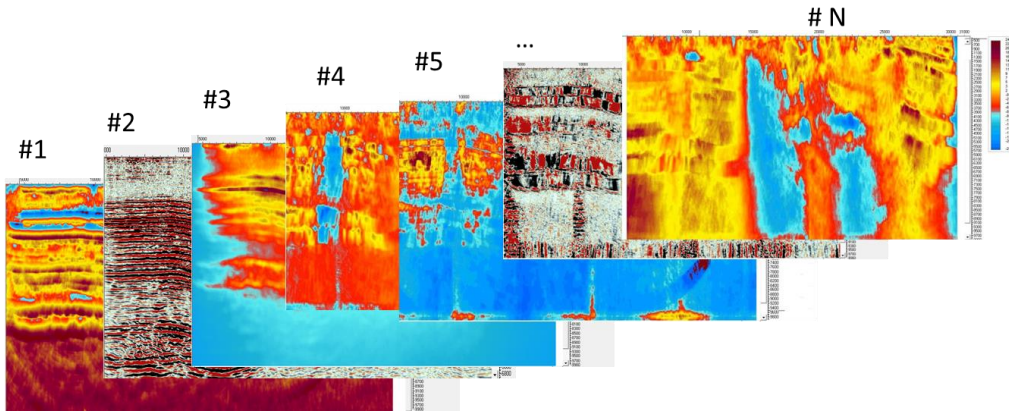


HPC is 50 Teraflops. 2D RTH velocity with GR curve and wiggle. The pixel size is 12.5x2.5 m. The dimension is 190x450 m. The red color of the palette is high-velocity deposits of Perm. Baltic Syncline, Curonian Depression.

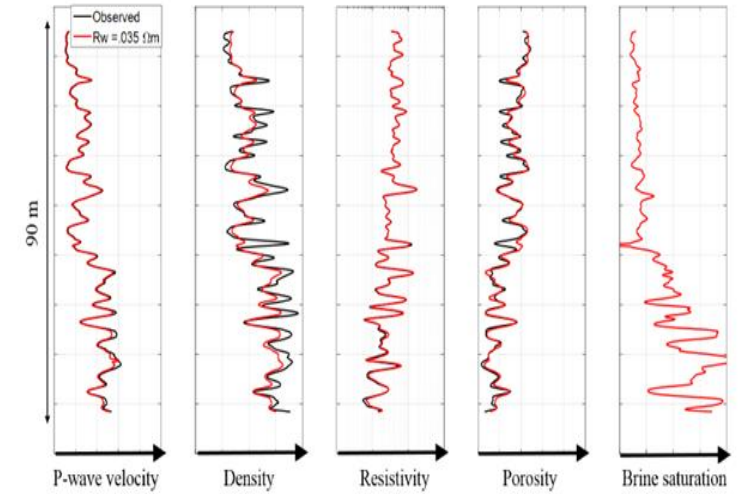


Forecast of hydrocarbon deposits based on RTH attributes and well logging data

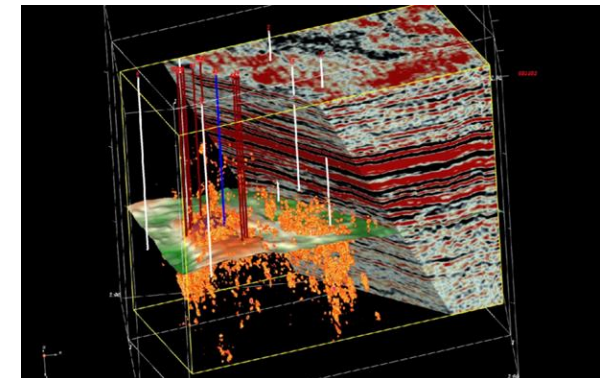
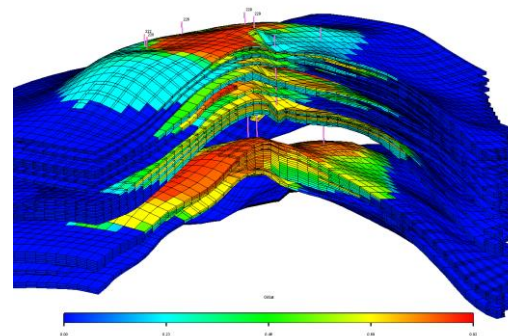
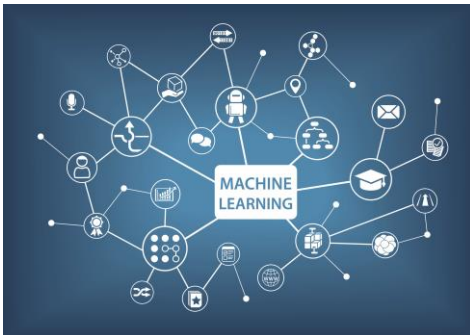
Synchronous calculation of RTH attributes in each voxel of a geological environment up to 1 meter in size



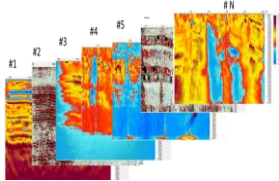
Geophysical well logging data



Machine learning to predict hydrocarbon deposits in the entire geological environment

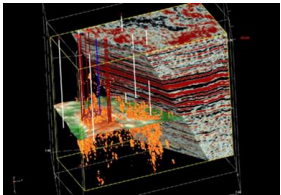


Practical possibilities of the RTH method



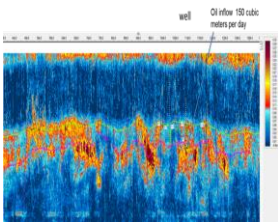
Providing seismic comparisons with downhole measurements

- abnormally high pressure zones
- non-structural hydrocarbon deposits
- direct forecast of hydrocarbon deposits



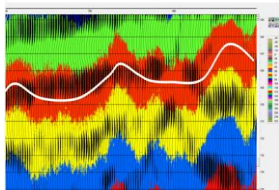
Audit of oil fields with complex geological structure

- identification of decompression zones in target horizons, open fracturing associated with faults
- permeability detection
- identification of areas with improved reservoir properties
- 4D seismic monitoring



Search for non-structural hydrocarbon deposits

- search for porous-fractured reservoirs
- substitution zone search

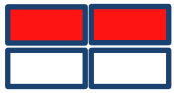


Seismic support for horizontal drilling

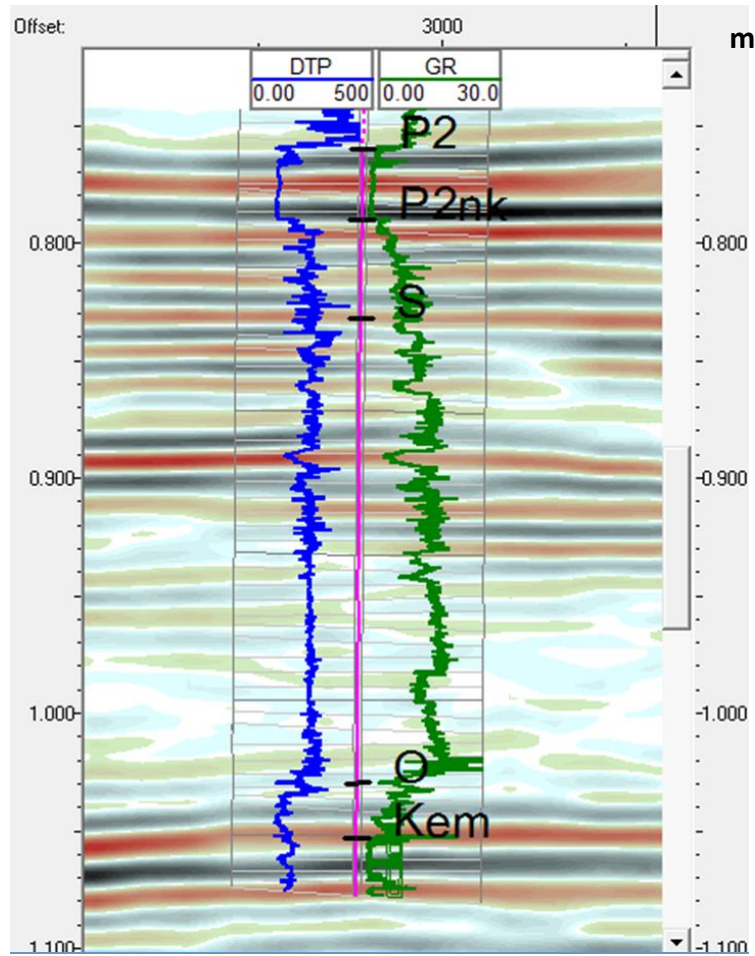
- building a high spatial resolution velocity model
- identification of areas with enhanced drainage properties

The list of advantages of the RTH method

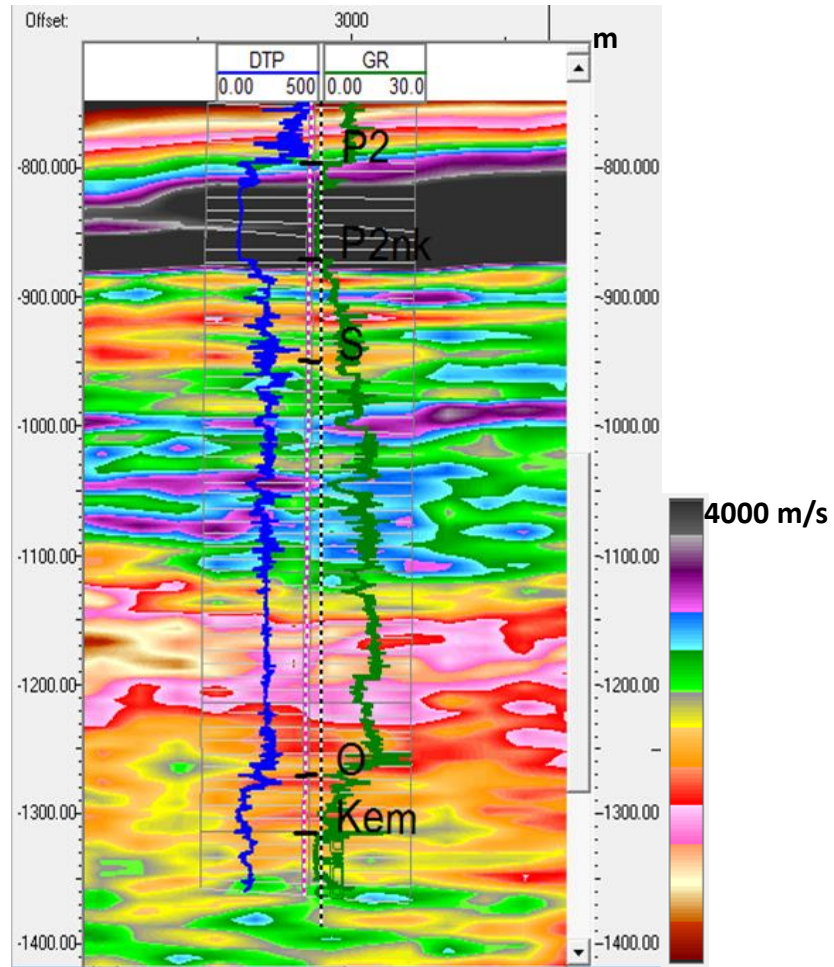
- Representation of the studied geological environment in the form of a set of volumetric cells (voxels) of arbitrary size with the subsequent calculation of the attributes in each cell independently of each other
- The spatial resolution of migration images is 3-6 times higher compared to conventional depth migration before summation
- Velocity tomography with a spatial resolution of up to 1 meter
- Lack of near-surface layer problem
- Stability RTH to sparse regular and sparse irregular seismic systems of receivers and sources
- Simultaneous and independent calculation of all known seismic attributes, such as: RTM, AVO, Dip, Opening Angle, azimuthal and spatial scattering anisotropy and even more than 50 new, previously unknown attributes



Case Study #1: Improving accuracy based on previously collected data



PSTM

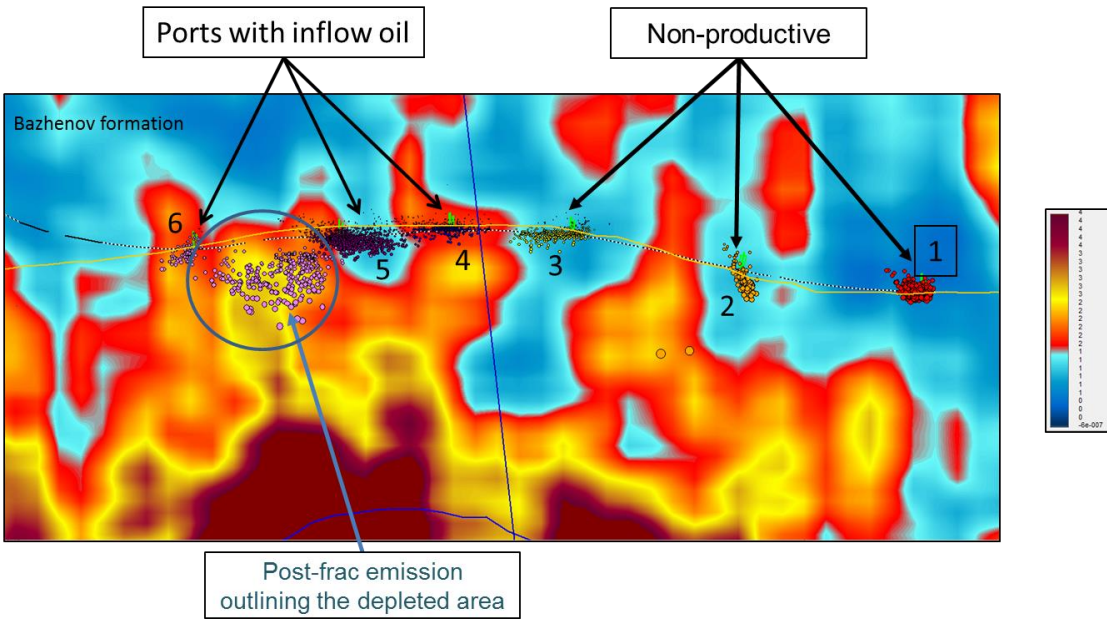


RTH Velocity

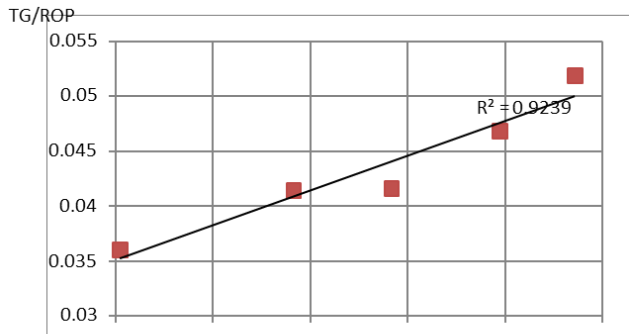
Comparison of PSTM and RTH velocity, in the area of a vertical well. The dark color of the palette is high-speed deposits of Perm. Baltic Syncline, Curonian Depression.



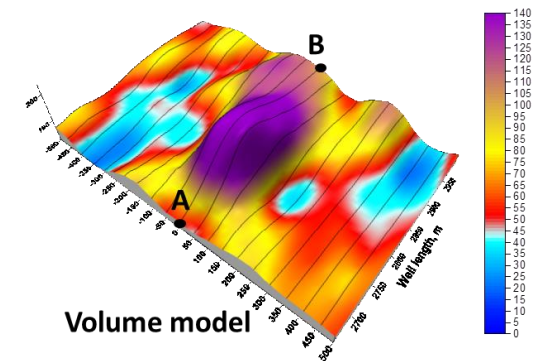
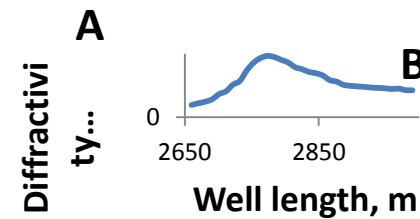
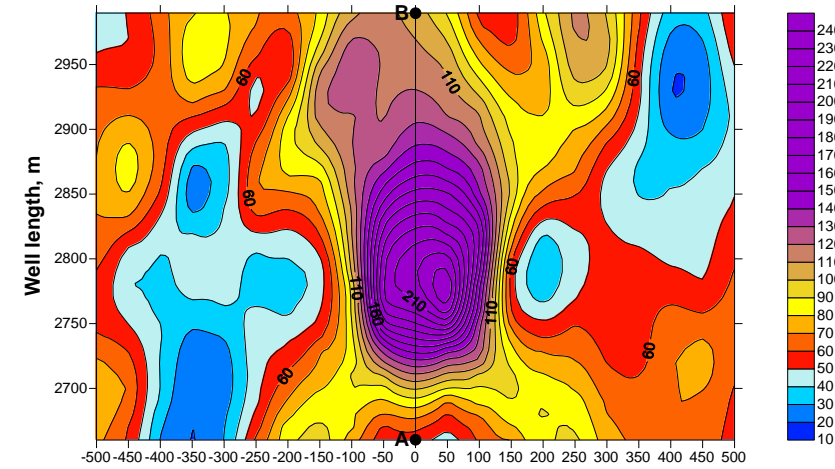
Case Study #2: Providing geosteering for horizontal well drilling



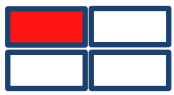
A vertical section of a fracture cube (sweet spot) along a horizontal wellbore. Ports ## 1-3 are dry, ports ## 4-6 are with oil. Western Siberia.



Correlation between fracture (sweet spot) and productivity based on horizontal well statistics.

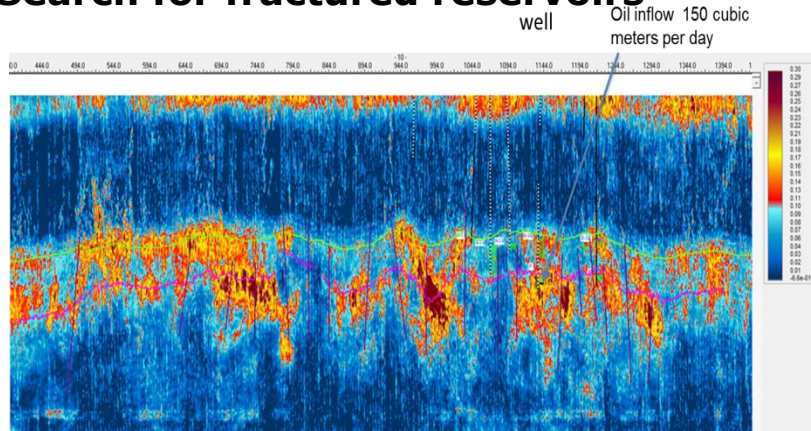


A horizontal section of a fracture cube along a horizontal well (A-B). Gas field. Eastern Siberia.



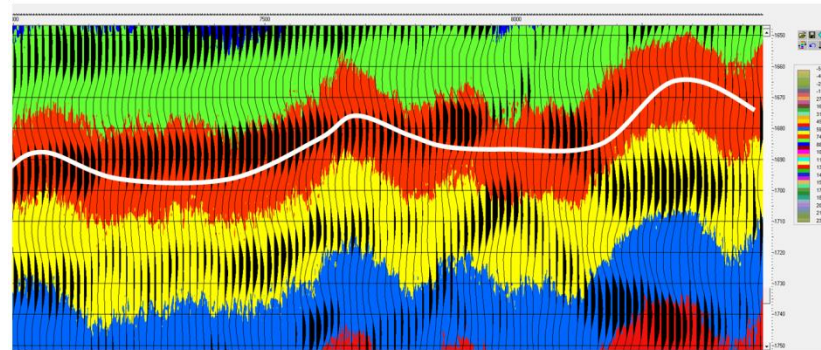
Case Study #3: High precision horizontal drilling support

Search for fractured reservoirs

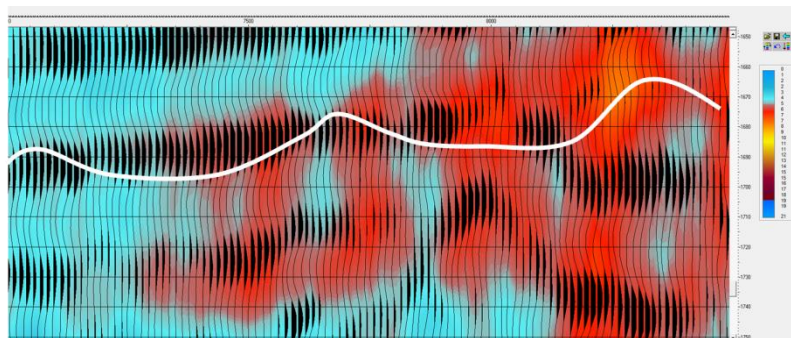


Fracture zones according to the RTH Scattering Index attribute with abnormally high oil production rates in the basement roof. The green curve is bazhen. The area size is 23x3 km. Western Siberia.

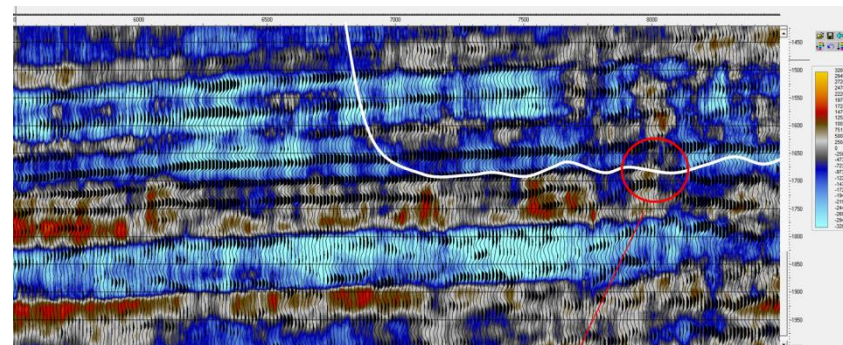
Seismic base for high precision geosteering in horizontal well drilling



RTH velocity attribute. The size of the area is 3000x100 m. The depth resolution is 2 meters. Arkhangelsk region.

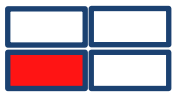


Fracture zones according to the RTH Scattering Index attribute for horizontal drilling. The size of the area is 3000x100 m. The depth resolution is 2 meters. Arkhangelsk region.

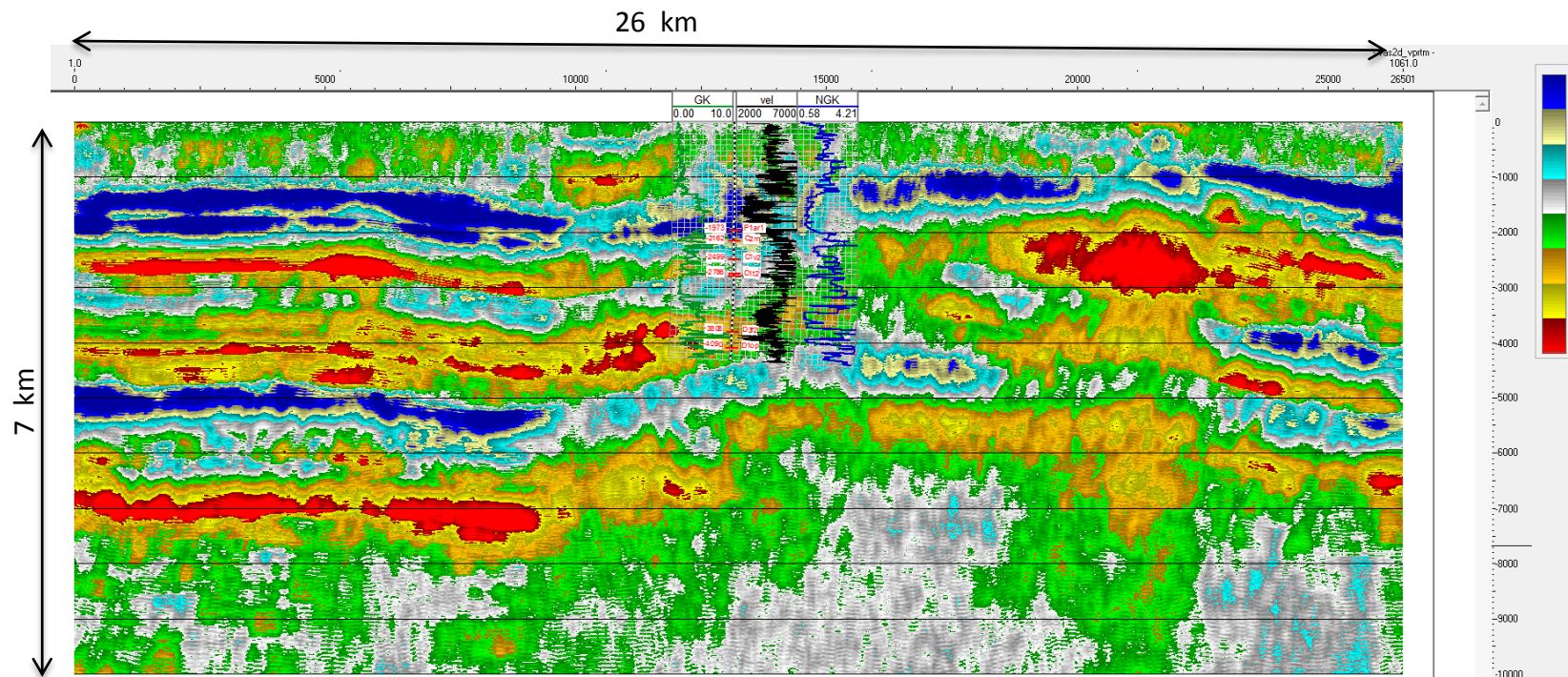


Change of Properties

RTH AVO Poison attribute. The size of the area is 6000x600 m. The depth resolution is 2 meters. Arkhangelsk region.

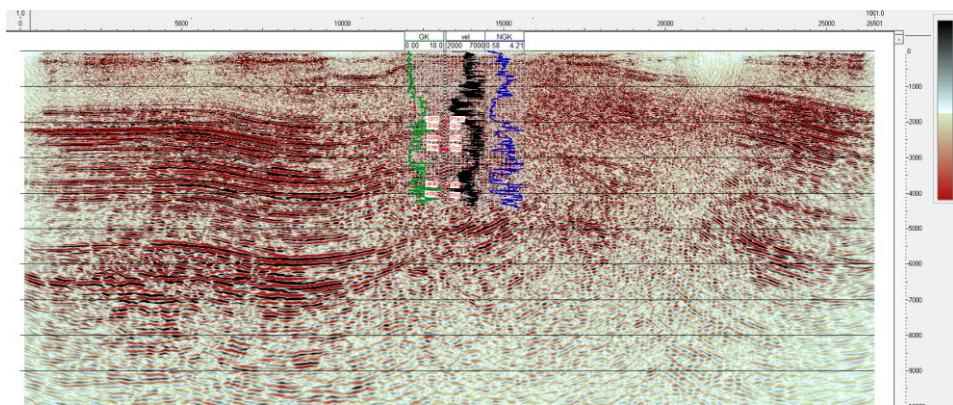


Case Study #4: Localization of regional decompression zones

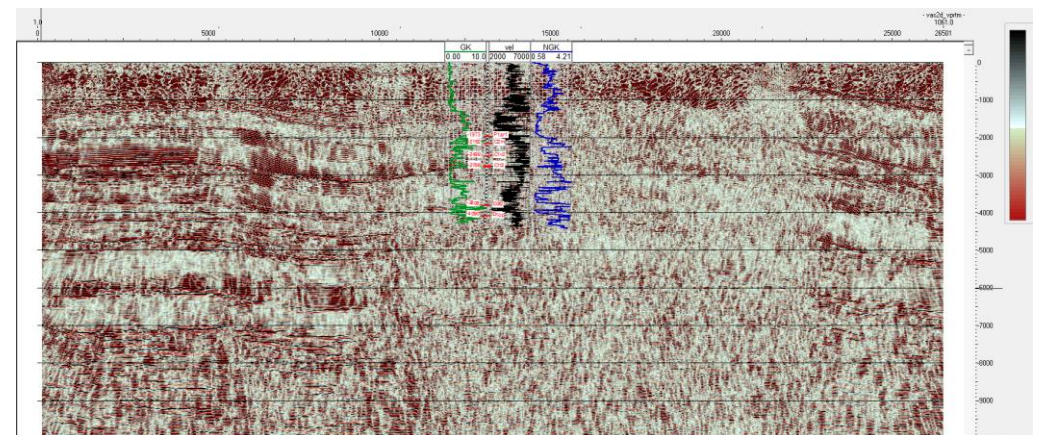


(a) Zones of lower RTH velocity in the upper part of the section (blue color) (b) Conventional PSDM (c) RTH Depth Migration. Well logging data from left to right: GR, Velocity, NGR. Arkhangelsk region.

a



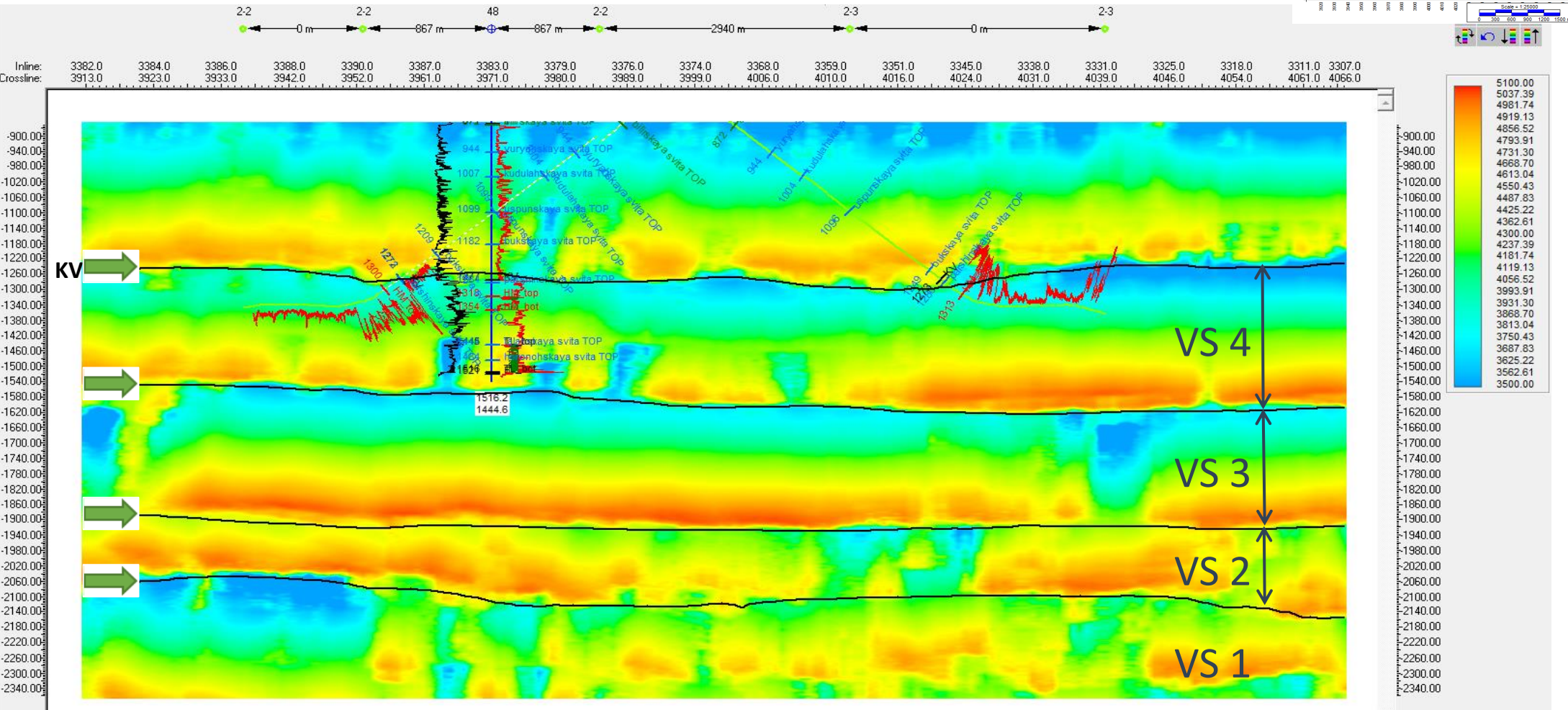
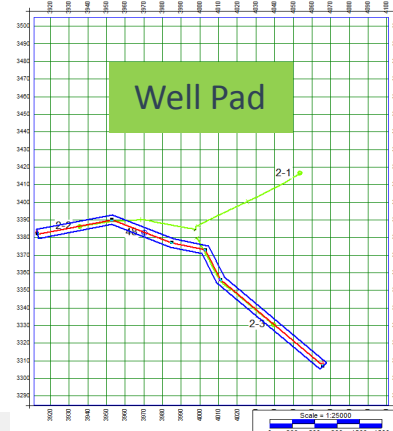
b



c



Case Study #5: RTH velocity stratum boundaries: georeferencing the RTH-velocity cube according the well logging data

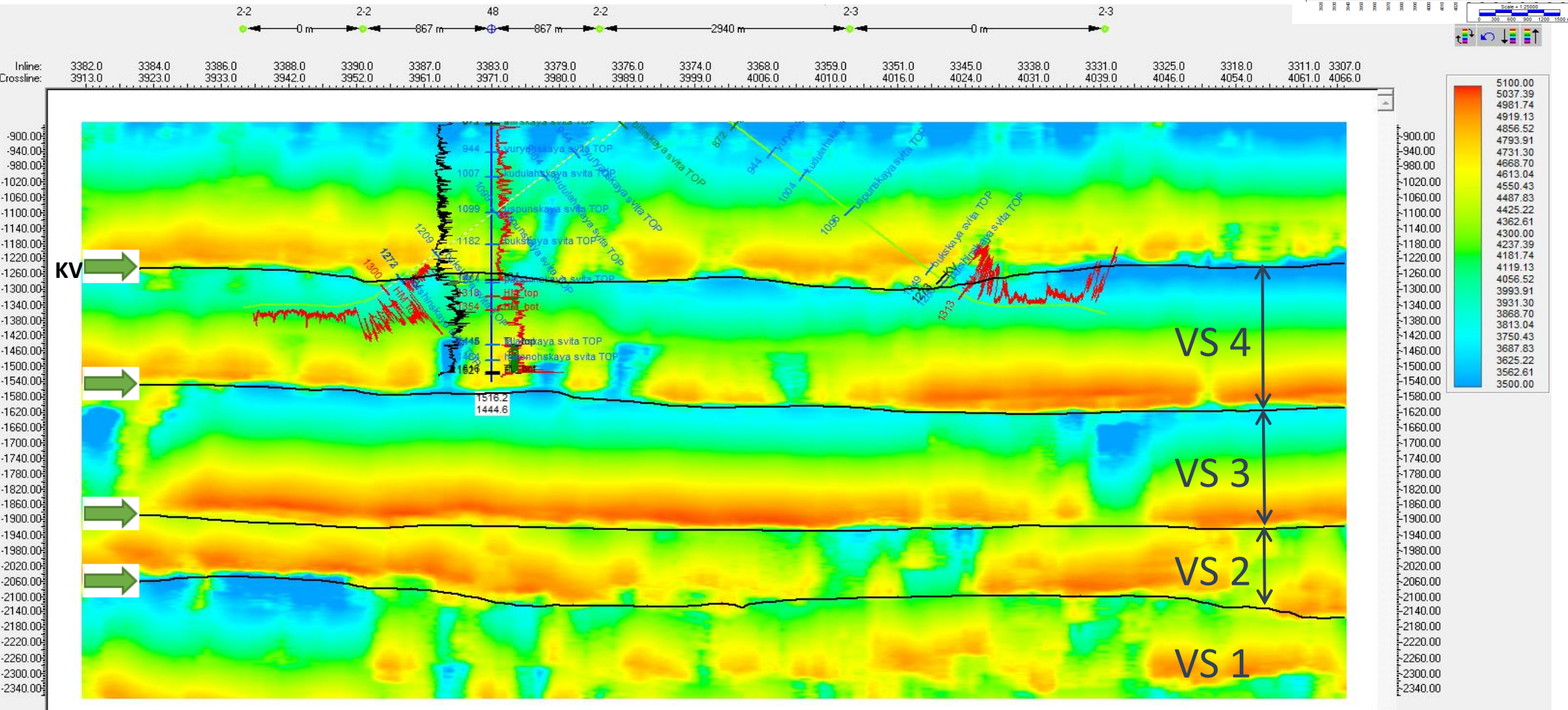
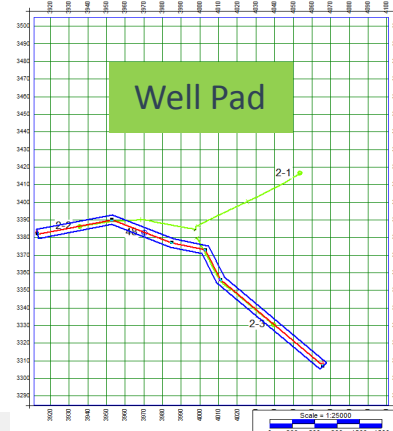


RTH-velocity scale, m / s

Voxel size is 12,5x12,5x2,5m



Case Study #5: RTH velocity stratum boundaries: georeferencing the RTH-velocity cube according the well logging data

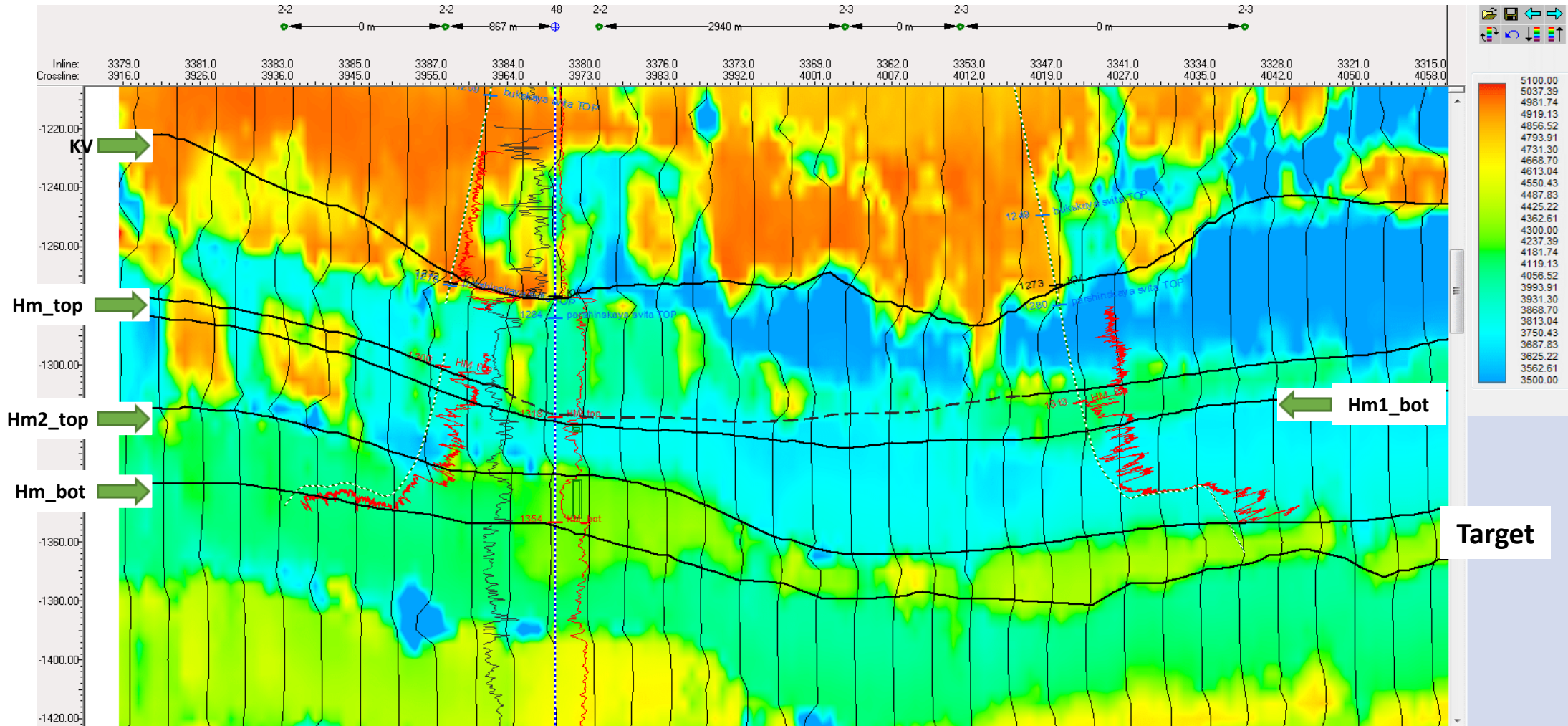
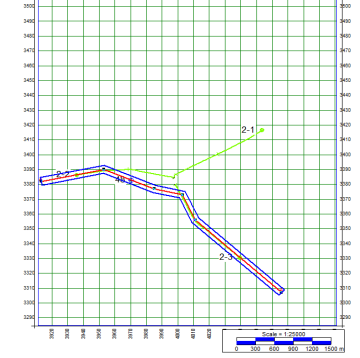


RTH-velocity scale, m / s

Voxel size is 12,5x12,5x2,5m

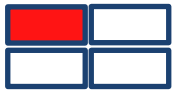


Case Study #5: RTH velocity-based mapping the productive upper (Hm1) and lower (Hm2) sub-horizons in Hm horizon

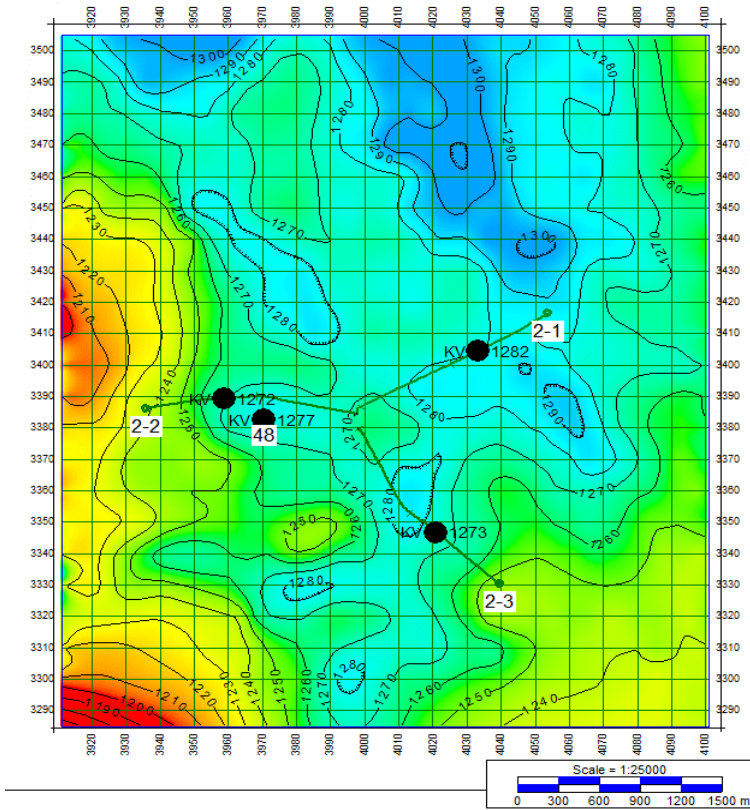


Scale RTH-velocity m/s, wiggle- RTH-velocity.

Voxel size is 12,5x12,5x2,5m

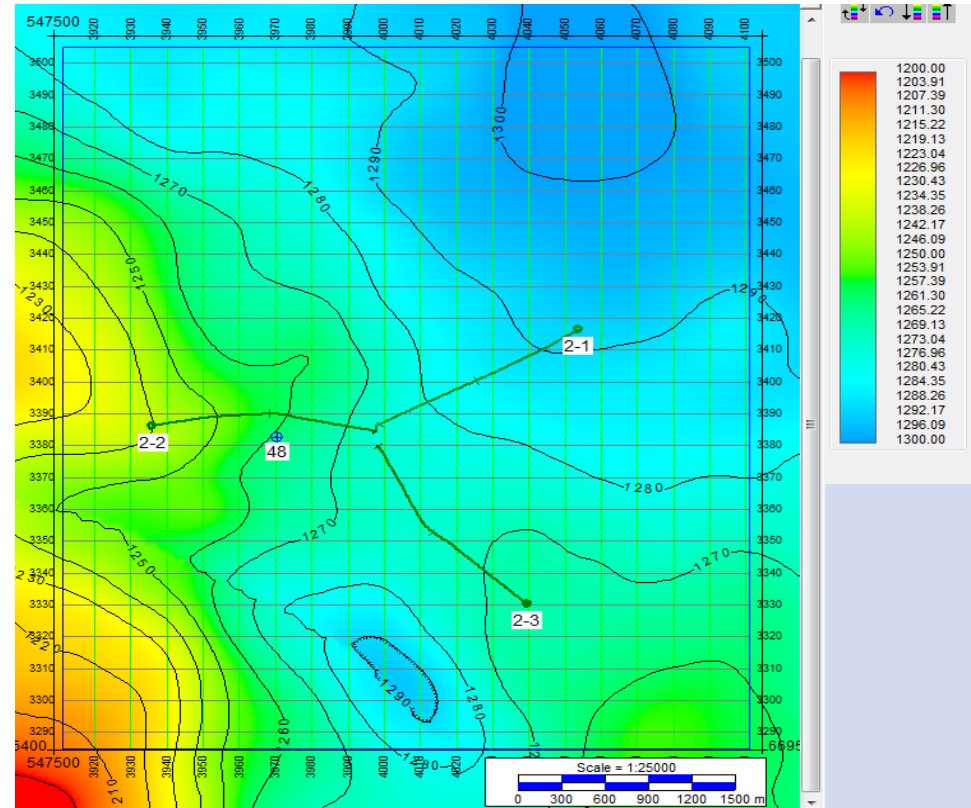
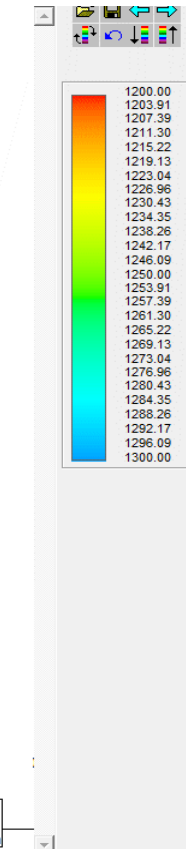


Case Study #5: Comparison the KV structural map constructed by velocity-based RTH approach with the conventional PSDM map



RTH map, m

Black dots on the map -
KV's depth by inclinometry



PSDM map, m

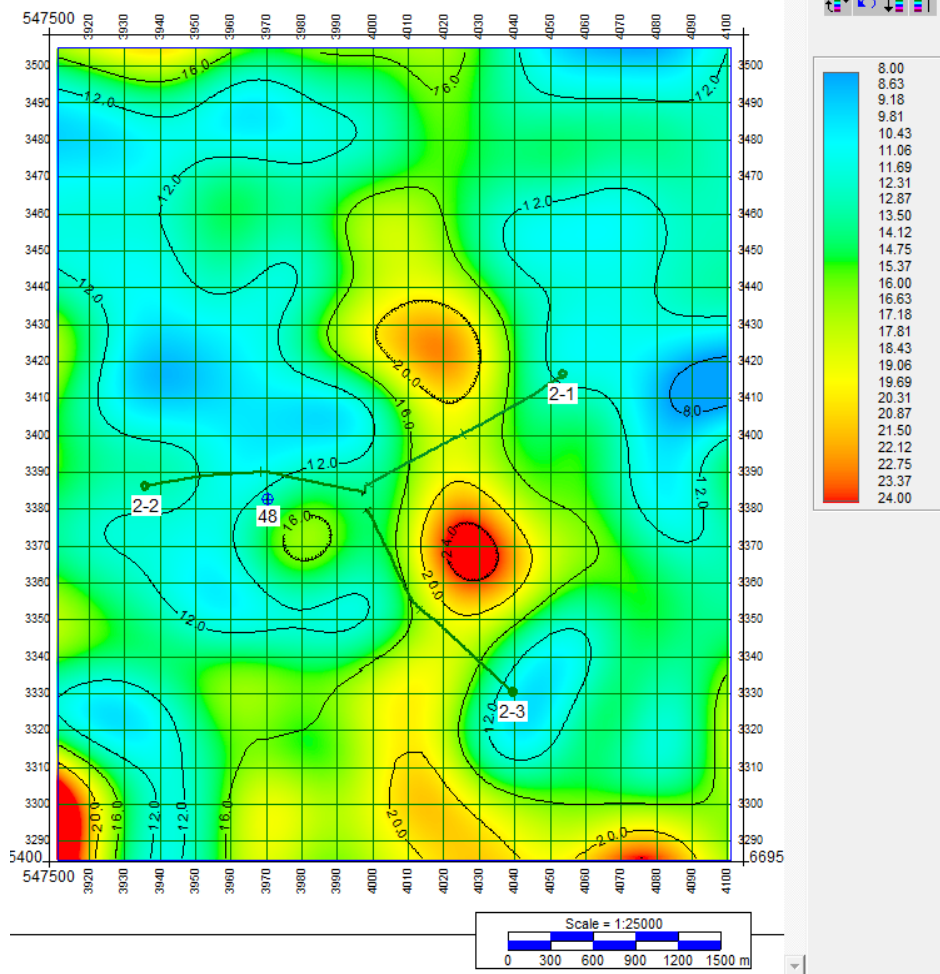
Map size 4x4 km

Voxel size is
12,5x12,5x2,5m

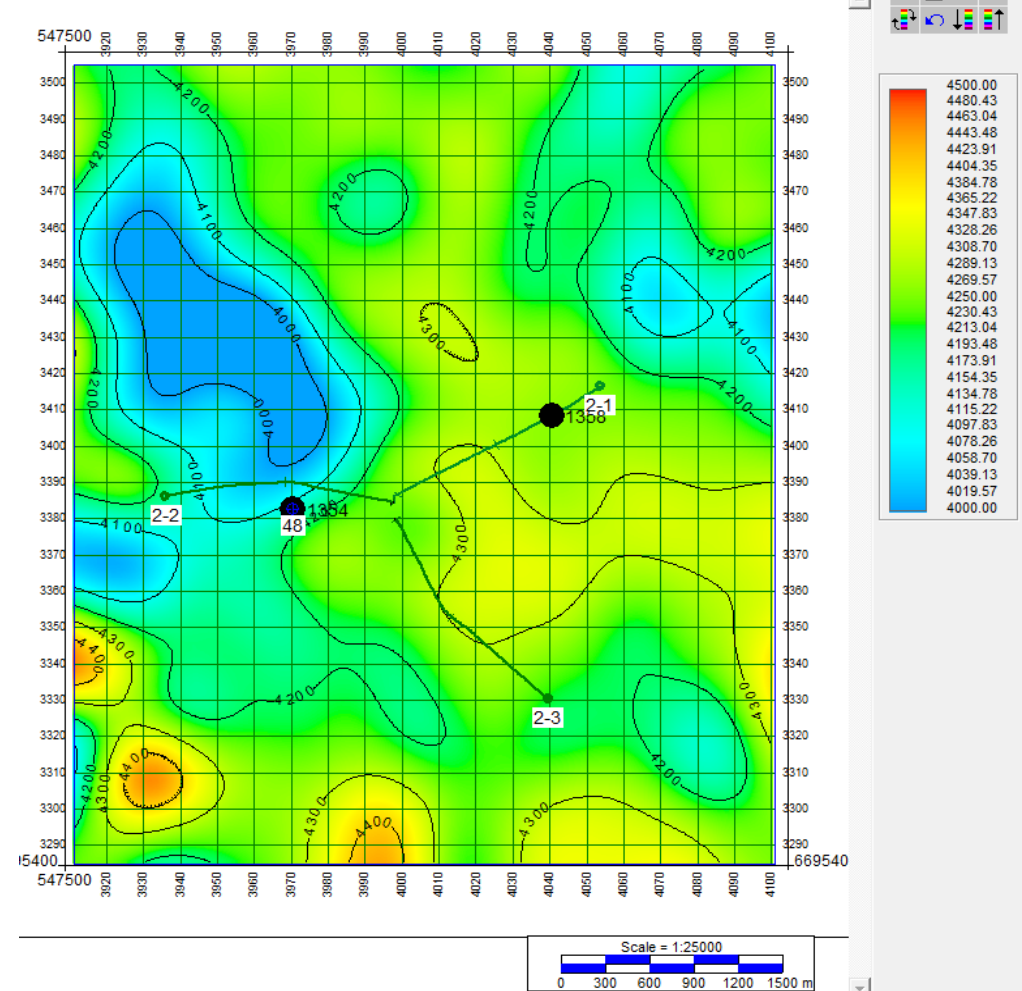
Well #	Depth, well (m)	Depth, RTH map (m)	Error (m)
48	1277	1272	5
2-1	1282	1280	2
2-2	1272	1271	1
2-3	1273	1274	1



Case Study #5: RTH velocity-based mapping the productive Hm2 horizon



Thickness map, meters



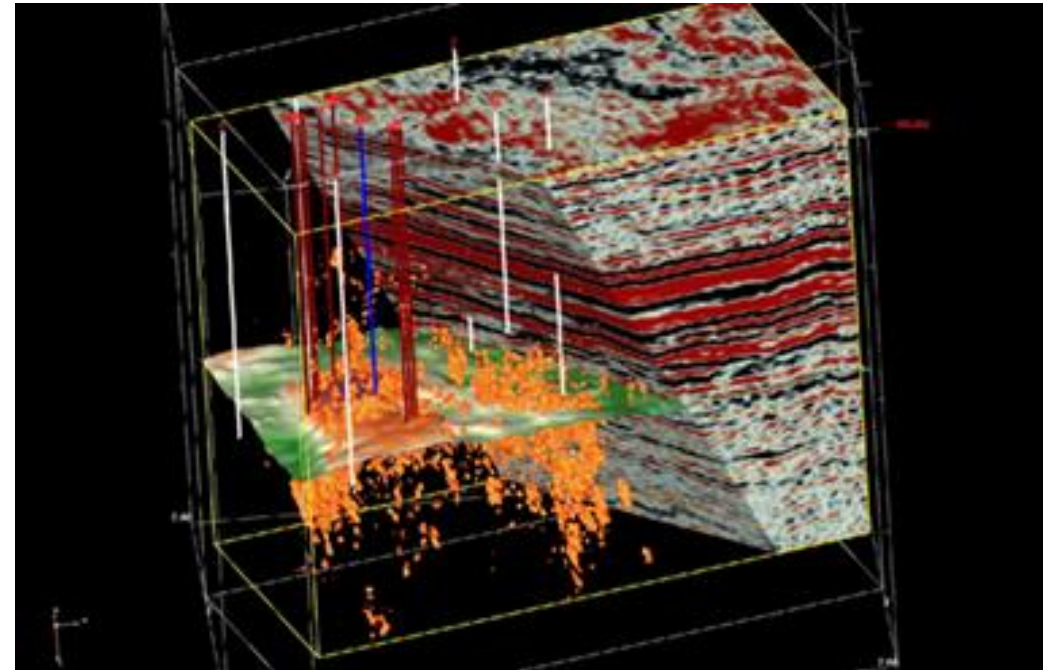
Velocity map, meters per second

Map size 4x4 km

Voxel size is
12,5x12,5x2,5m

Conclusions

1. The RTH method, as in optical holography, “accurately” captures information about the amplitudes and phases of a scattered and time-reversed seismic wave using some “reference” wave — the stage of decomposition of the RTH method.
2. This information is stored in the VDCIG repository (Vector Domain Common Image Gathers). The construction of seismic attributes is based on a direct statistical estimation of the dataset from VDCIG - the stage of synthesis of the RTH method.
3. The RTH method allows you to simultaneously obtain various voxel-based attributes of high spatial resolution (up to 1 meter) such as reflectors, diffractors, dip angles, scattering anisotropy, azimuthal anisotropy, AVO, etc.
4. The method provides velocity tomography with ultra high spatial resolution.
5. The RTH method requires more processing power than the conventional RTM depth migration method, but significantly less than the high frequency FWI methods.
6. Method RTH is stable to sparse regular and sparse irregular seismic systems of receivers and sources and so special RTH seismic more cost effective than conventional seismic
7. Near-surface layer problem is absent for RTH method



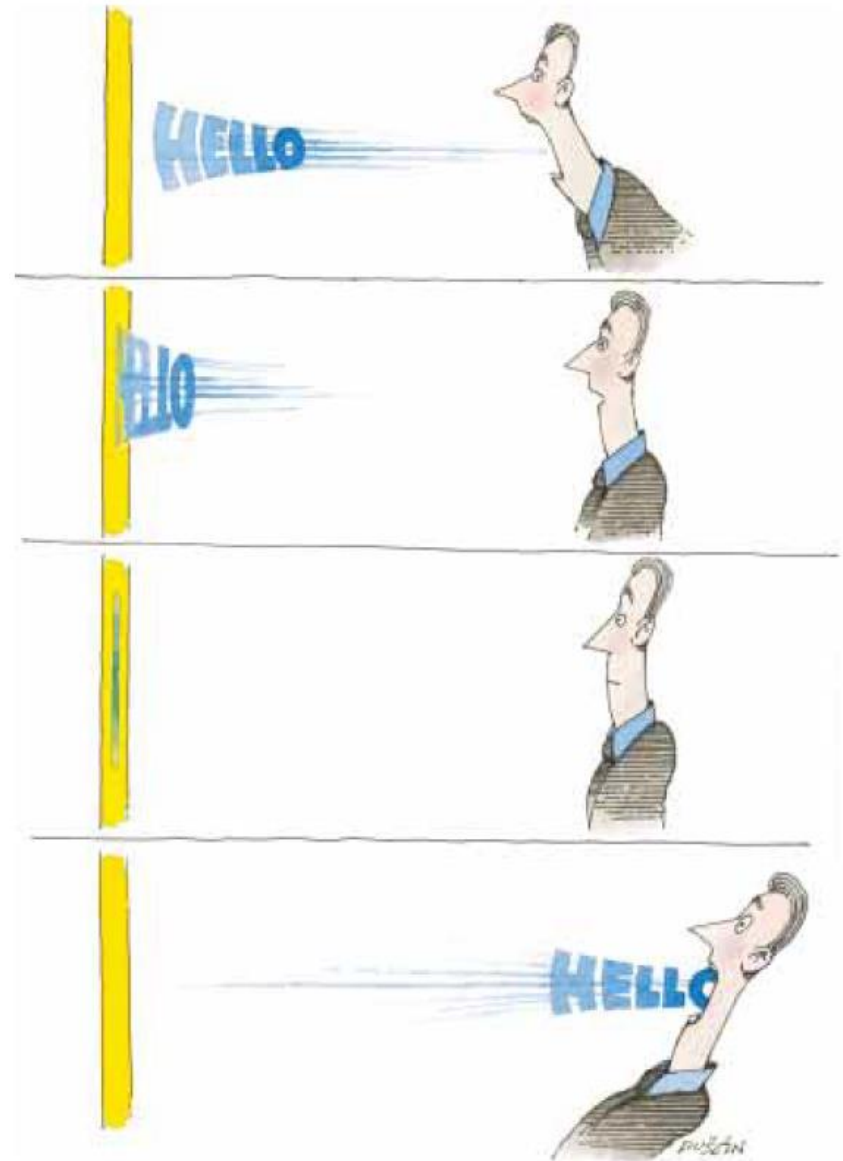
Reflected and scattered wave field components

Publications

1. Gennady Erokhin and Vitaly Bryksin, High-resolution velocity model estimation by the RTH method, 2020, SEG Technical Program Expanded Abstracts, 2020: 2863-2867 <https://doi.org/10.1190/segam2020-3410422.1>
2. Gennady Erokhin, New Possibilities of the Interpretation Seismic and Logging Data by Machine Learning Based on the RTH's Attributes, 2020, SEG AI Earth Exploration Workshop: Teaching the Machine How to Characterize the Subsurface, 23-26 November 2020, Oman (in print)
3. E.V. Anokhina, V.M. Bryksin, G.N. Erokhin and V.K. Chegesov, Prospects for Expanding the Hydrocarbon Base on the Basis of Complex New Seismic Methods and Well Data, 2020, European Association of Geoscientists & Engineers, Conference Proceedings, Geomodel 2020, Sep 2020, Volume 2020, p.1 – 5 DOI: <https://doi.org/10.3997/2214-4609.202050089>
4. V. Bogoyavlensky, R. Nikonov, G. Erokhin and V. Bryskin, Passive Seismic Monitoring Study of the Earth Degassing in the Arctic, 2020, European Association of Geoscientists & Engineers, Conference Proceedings, Geomodel 2020, Sep 2020, p.1 – 5 , DOI: <https://doi.org/10.3997/2214-4609.202050102>
5. Gennady Erokhin, Reverse Time Holography Approach based on the Vector Domain Common Image Gathers, 2019, SEG Technical Program Expanded Abstracts 2019: 4107-4111., <https://doi.org/10.1190/segam2019-3201622.1>
6. Gennadiy Erokhin, Alexander Bugaev, Maksim Kozlov, Alexander Danilin. The Use of Parallel Technologies in the Solution of a Problem of Geophysics Concerned with the Detection of Weakly Scattering Objects//In: Sokolinsky L., Zymbler M. (eds) Parallel Computational Technologies. PCT 2019. Communications in Computer and Information Science, vol 1063, pp.185-196(2019)(Scopus). https://link.springer.com/chapter/10.1007%2F978-3-030-28163-2_13
7. Anokhina E. and G. Erokhin The Use of New Seismic Data Processing Methods to Predict Changes in Reservoir Properties During the Design of Horizontal Wells, 2019, EAGE, Horizontal Wells 2019 Challenges and Opportunities. <https://doi.org/10.3997/2214-4609.201901854> (in Russian)
8. Erokhin G. , Bugaev A., Bogoyavlesky I., Prospects for combining a new method of seismic exploration RTH and the results of passive microseismic monitoring in solving the problems of identifying dangerous objects of gas emissions in the Arctic, 2019, Drilling and oil 07-0 8/2019, p. 4-9. 4.
9. Erokhin G., Danilin A. and M. Kozlov, Visualization of Ultra-Weak Diffractors based on Vector Pair RTM, 2018, 80th EAGE Conference and Exhibition 2018, <https://doi.org/10.3997/2214-4609.201801648>
10. Erokhin G., Danilin A., and Kozlov M., Extension of the common image gathers by VPRM method, 2018, SEG Technical Program Expanded Abstracts 2018: pp. 4438-4442. <https://doi.org/10.1190/segam2018-2995971.1>
11. Erokhin G., Pestov L., Danilin A., Kozlov M., and Ponomarenko D., Interconnected vector pairs image conditions: New possibilities for visualization of acoustical media, 2017, SEG Technical Program Expanded Abstracts 2017: 4624-4629., <https://doi.org/10.1190/segam2017-17587902.1>

THANKS!

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The essence of the RTH method

The essence of the RTH method - Reverse Time Holography is the numerical implementation for the seismic of the physical effect of the "reversing mirror" open for laser, B.Ya. Zeldovich (Lebedev Physical Institute of the USSR) in 1972. Implementation is based on the theory of conjugate equations.

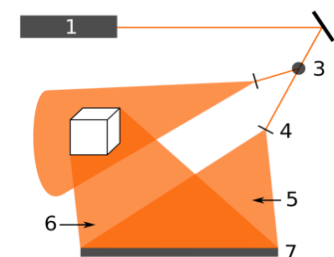
Unlike a conventional reflecting mirror, in which ordering of reflections in time is performed (the pulse that arrived first - is reflected first), the optical or wavefront, after interacting with a "reversing mirror", turns in time (Reverse Time: first came - last reflected) .



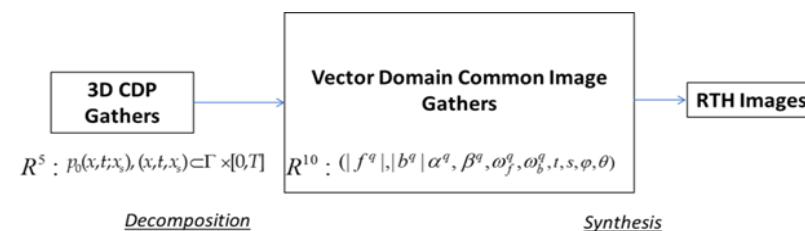
Moreover, the time-reversed wave front reflected from the reversing mirror "remembers" the turbulence or inhomogeneity of the medium through which the wave passed before reaching the reversing mirror and, when passing through the same medium in the opposite direction, the reversed wavefront is self-correcting, that is, it is cleared from these primary distortions.

In the case when the primary seismic wave field incident on the "reversing mirror" was already scattered by the inhomogeneities of the geological medium during its primary passage, after reversal and passage through the inhomogeneous medium in the opposite direction, the undistorted corrected wave field will be precisely focused at the points of primary scattering . In seismic, such a primary scattered wave field is generated by backscattering of the medium from a wave generated by a source located on the surface of the Earth.

The focusing process in space and time in the RTH method is fixed using the interferometry method, which is used in holography for the spatial interaction of two waves: reference and scattered.



The role of the holographic photographic plate in the RTH method is performed by the Vector Domain Common Image Gather -VDCIG digital storage of 4-dimensional data, which stores all the amplitude and phase information about the time-reversed seismic wave scattered through the inhomogeneities of the medium.

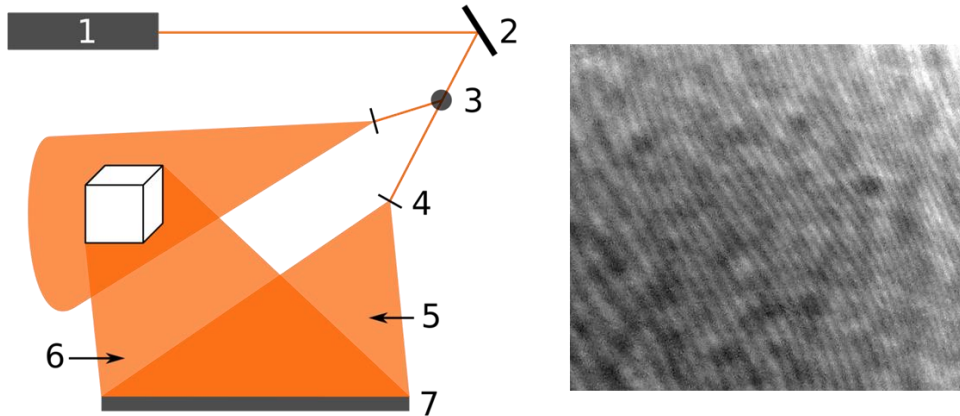


Improving the accuracy of information in VDCIG is ensured by multiple generation of the seismic process at various points on the Earth's surface. The formation of the VDCIG data repository in the RTH method is similar to the procedure of the full spatio-temporal "decomposition" of a scattered wave.

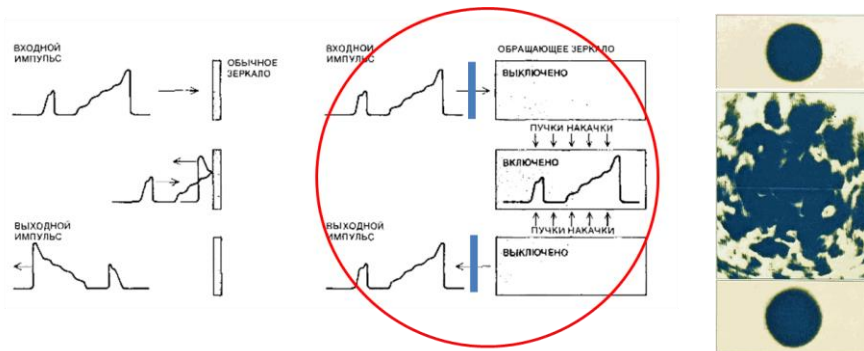
Evaluation of the properties of the heterogeneous medium (velocity, impedance, reflectivity, scattering indicatrix, etc.) based on VDCIG data is the essence of the second stage of RTH, the "synthesis" stage.²⁵

Prerequisites for creating the RTH method

1. Optical holography

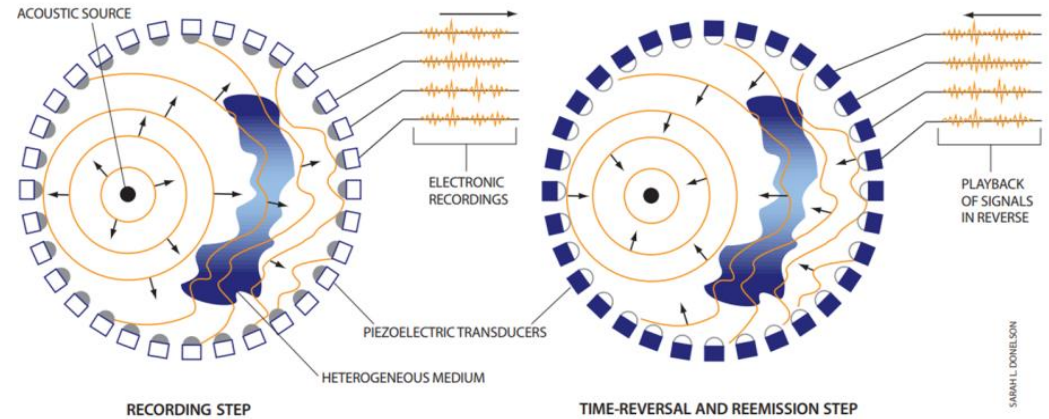


2. Reversing mirror for laser



Zel'dovich B.Ya., Popovichev, Ragulsky VV, Fayzullov FS, 1972, On the relationship between the wavefronts of reflected and exciting light in stimulated Mandelstam-Bryullen scattering. Letters to JETP, vol. 15, No. 3 pp. 160-164.

3. Acoustic wave time reversal. Mathias Fink



ACOUSTIC TIME-REVERSAL MIRROR operates in two steps. In the first step (left) a source emits sound waves (orange) that propagate out, perhaps being distorted by inhomogeneities in the medium. Each transducer in the mirror array detects the sound arriving at its location and feeds the signal to a computer.

In the second step (right), each transducer plays back its sound signal in reverse in synchrony with the other transducers. The original wave is re-created, but traveling backward, retracing its passage back through the medium, untangling its distortions and refocusing on the original source point.

4. Adjoint equations

Forward wave $(p^f, \vec{u}^f)(x, t; x_s), t \in [0, T]$

$$p_t^f - c^2 \operatorname{div}(\vec{u}^f) = r(t)\delta(x - x_s) \quad (1)$$

$$\vec{u}_t^f = \nabla p^f$$

$$p^f|_{t=0} = 0, \quad \vec{u}^f|_{t=0} = 0.$$

$$x_s \in \Gamma = \{x \in \mathbb{R}^n \mid x^n = 0\}, t \in (0, T)$$

Back wave $(p^b, \vec{u}^b)(x, t; x_s), t \in [0, T]$ $p_0 = p^f|_{\Gamma \times [0, T]}$

$$p_t^b - c^2 \operatorname{div}(\vec{u}^b) = 0 \quad (2)$$

$$\vec{u}_t^b = \nabla p^b + p_0 \delta(x^n) \vec{\nu}_\Gamma$$

$$p^b|_{t=T} = 0, \quad \vec{u}^b|_{t=T} = 0,$$

$$\begin{cases} f = \vec{u}^f \\ b = \vec{u}^b \end{cases}$$

RTH background

The technique of integration by parts underlies the most important mathematical approaches, such as the definition of Lagrange conjugate operators, Bayes' formulas, conjugate problem statements for differential equations etc. As it turned out, in physics, conjugate processes also underlie a number of known technologies such as time-reversal mirror for laser beam (Zel'dovich et.al.,1972) and time-reversed acoustics (Fink, 1997).

In seismic prospecting, conjugate formulations for the wave equation are the basis of the well-known Reverse Time Migration (RTM) method (Baysal et.al., 1983; McMechan, 1983). In Full Wave Inversion method, conjugate mathematical formulation is used to calculate the Frechet derivative while minimizing misfit functional (Tarantola, 1984; Virieux, et.al., 2009).

Alekseev and Erokhin, 1989 for the first time, mentioned the close connection between the essence of the conjugate approach for Simultaneous Joint Inversion (SJI) method, proposed by the authors and of time-reversal mirror for laser beam. They also marked the mathematical similarity of the SJI approach with the optimal control problem in ecology (Marchuk, 1976). In the same paper, the convergence of JSI solution on some weakly compact set and an increase in the stability of the solution on it were constructively proved.

In the papers Erokhin, 2019 a new Reverse Time Holography (RTH) method is proposed for design seismic attributes. This method combines two approaches: reversal of a wave in time based on the conjugate problem for the acoustic equation and two-beam interferometry, similar to that used in optical holography (Gabor, 1947).

The data processing consists of two stages: decomposition and synthesis (Erokhin et.al., 2020). Decomposition stage includes a highly accurate vector decomposition of time-reversed seismic information which is recorded at the surface, for example by CDP method. At this stage, as in optical holography, a certain reference wave is also used for two-beam interferometry. The received information about the interference of the reference wave and time-reversed wave forms a set of vector pairs, which is called Vector Domain Common Image Gathers (VDCIG), (Erokhin et.al., 2018a). This digital dataset is similar a photographic plate for optical holography, on which the amplitudes and phases of two-beam interference are recorded.

The second stage of RTH (synthesis) consists in statistical evaluation of the parameters of the multidimensional random distribution of VDCIG in order to obtaining the necessary seismic attributes. It turns out that such a formal mathematical approach makes it possible to construct by RTH method not only all known seismic attributes, but also to obtain much of new ones (Erokhin, 2019).

References

- Alekseev A.S., and Erokhin G.N., 1989, Integration in geophysical inverse problems (Integrated Geophysics), USSR Academy of Sciences Proceedings, Volume 308. № 6., UDC 550.3:517.97, p.1327-1331, <http://rthtech.com/articles/>
- Baysal, E., D. D. Kosloff, and J. W. C. Sherwood, 1983, Reverse time migration: Geophysics, 48, 1514-1524, <https://doi.org/10.1190/1.1441434>
- Erokhin G., Pestov L., Danilin A., Kozlov M., and Ponomarenko D., 2017, Interconnected vector pairs image conditions: New possibilities for visualization of acoustical media, 2017, SEG Technical Program Expanded Abstracts 2017: 4624-4629., <https://doi.org/10.1190/segam2017-17587902.1>
- Erokhin Gennady, Danilin Aleksandr, and Maksim Kozlov, 2018a, Extension of the common image gathers by VPRTM method. SEG Technical Program Expanded Abstracts 2018: pp. 4438-4442.
- Erokhin G., Danilin A. and M. Kozlov, 2018b, Visualization of Ultra-Weak Diffractors based on Vector Pair RTM, 80th EAGE Conference and Exhibition 2018, doi: 10.3997/2214-4609.201801648
- Erokhin G., Reverse Time Holography Approach based on the Vector Domain Common Image Gathers, 2019, SEG Technical Program Expanded Abstracts 2019: 4107-4111., <https://doi.org/10.1190/segam2019-3201622.1>
- Erokhin Gennady and Vitaly Bryksin, High-resolution velocity model estimation by the RTH method, 2020, SEG Technical Program Expanded Abstracts, 2020: 2863-2867 <https://doi.org/10.1190/segam2020-3410422.1>
- Fink Mathias, 1997, Time Reversed Acoustics,. Physics Today. 50 3: 34. doi:10.1063/1.881692.
- Gabor, D. A new microscopic principle. Nature 161, 777_778 (1948).
- Marchuk G.I.. 1976. Academy of Science Proceedings. V. 227, № 5. 1056-1059
- McMechan, G. A., 1983, Migration by extrapolation of time-dependent boundary values: Geophysical Prospecting, 31, 413-420, doi: 10.1111/j.1365-2478.1983.tb01060.x.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics, 49, 1259-1266. <http://dx.doi.org/10.1190/1.1441754>
- Virieux, J., and Operto, S., 2009, An overview of full-waveform inversion in exploration geophysics: GEOPHYSICS, 74, WCC1-WCC26. <http://dx.doi.org/10.1190/1.3238367>
- Zel'dovich B.Ya, V.I.Popovichev, V.V.Ragulsky, F.S.Faizullof., 1972, On the connection between wavefronts of reflected and exciting light in stimulated scattering of Mandelstam-Brüllen. Letters to ZhETF, v. 15, no. 3 p. 160-164